TeachME Professional Development

Improving Mathematical Problem Solving-2018 Update

Introduction

1. Experts believe that students who develop proficiency in mathematical problem solving early are better prepared for advanced mathematics and other complex problem-solving tasks later in life.

A. True
B. False

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2. Mathematical problem solving involves reasoning and analysis, argument construction, and:

A. Verbalization of thought processes
B. Corrective feedback
C. Guided practice
D. The development of innovative strategies

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3. The practice guide expert panel defined the characteristics of problem solving to include each of the following EXCEPT:

A. Students can learn mathematical problem solving, as it is neither an innate talent nor happenstance that creates skilled problem solvers
B. Mathematical problem solving need not be treated like just another topic in the pacing guide, but instead, it can serve to support and enrich the learning of mathematics concepts and notation
C. Mathematical problem solving is relative to the group for which it is being taught, and what is challenging or non-routine for one student is generally the same for the group
D. Often more than one strategy can be used to solve a problem, and may enable students to think more flexibly when presented with a problem that does not have an obvious solution

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Summary of the Recommendations-Teaching Problem Solving

4. Experts recommend teaching one strategy that can be used to solve multiple problems, as this will be less likely to overwhelm the students than teaching multiple strategies.

A. True
B. False

Table 2. Recommendations and Corresponding Levels of Evidence

5. Which of the following recommendations for math problem solving is supported by a strong level of research evidence?

A. Teach students how to use visual representations
B. Prepare problems and use them in whole-class instruction
C. Expose students to multiple problem-solving strategies
D. Help students recognize and articulate mathematical concepts and notation

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Prepare Problems and Use Them in Whole-Class Instruction

6. The expert panel believes instruction in problem solving must be an integral part of each curricular unit, and that teachers should use a variety of problems intentionally while ensuring that students have the language and mathematical content necessary to solve problems.

A. True
B. False

How to Carry out the Recommendation--Definitions of Routine and Non-Routine Problems

7. In order to determine which problems will be routine or non-routine for students, teachers must consider students':

A. Ability to retrieve basis math facts
B. Level of motivation
C. Previous experience with problem solving
D. Comfort with verbalizing the steps to solve a problem

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8. Cognitive demands are associated with solving non-routine problems, and generally

much more time will be needed to interpret the problem and determine what information is relevant, as well as how it should be used.

A. True
B. False

Addressing Issues Students Might Encounter with the Problem's Context or Language

9. Which of the following is NOT one of the recommendations for teachers in helping prepare lessons to ensure student understanding?

A. Choose problems with familiar contexts
B. Choose challenging problems that will increase students' confidence in problem solving
C. Clarify unfamiliar language and contexts
D. Reword problems, drawing upon students' experiences

Potential Roadblocks and Solutions

10. To make time during instruction for problem solving, teachers should consider balancing the number of problems students are required to solve during seatwork activities with worked examples students can simply study.

A. True
B. False

Summary of Evidence: Strong Evidence

11. Studies that examined students' mathematics achievement in different content areas showed that:

A. Providing students with a task list that identified specific steps to solving problems resulted in better student achievement
B. Using a self-questioning checklist improved achievement
C. Student performance improved when teachers modeled a self-questioning process and then asked students to practice it
D. All of the above

How to Carry out the Recommendation-Provide Students with a List of Prompts

12. Experts recommend that students be encouraged to explain and justify their response to each mathematical prompt orally rather than in written form, and preferably in a group setting rather than individually.

A. True
B. False

Example 7. Sample Task List

13. When responding to prompts, steps include identifying the givens and goals of the problem and the problem type, recalling similar problems to help solve the current problem, _____, solving the problem, and checking the solution.

A. Using a visual to represent and solve the problem
B. Distinguishing substantive information from superficial information
C. Identifying underlying structural features of each problem
D. Getting feedback from peers

Use Student Thinking About a Problem to Develop Students' Abilities

14. By building on students' ideas, teachers can help students clarify and refine the way they solve a problem, which may be particularly helpful for students who dislike working with teacher-provided prompts or who are having difficulty understanding these prompts.

A. True
B. False

Teach Students How to Use Visual Representations

15. A major task for any student engaged in problem solving is to translate the quantitative information in a problem into a:

A. Graphic illustration
B. Symbolic equation
C. Distinct diagram

D. Allegorical statement

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16. Schematic diagrams are visual representations that use rectangles to represent quantities presented in the problem.

A. True
B. False

Use Think-Alouds and Discussions to Teach Students how to Represent Problems Visually

17. Thinking aloud involves having the teacher tell students what he or she is doing, and expressing thoughts while explaining what decisions he or she is making and why those decisions are being made.

A. True
B. False

Example 12. Variations in Story Contexts for a Proportion Problem

18. Teachers may promote discussions by asking students guiding questions as they practice representing problems visually, such as each of the following EXCEPT:

A. What kind of problem is this and how do you know?
B. What is the relevant information in this problem and why is it relevant?
C. Which problem solving proficiencies did you use when you solved this type of problem last time?
D. What would you do next? Why?

Expose Students to Multiple Problem-Solving Strategies: Summary of Evidence: Moderate Evidence

19. The estimated effects of teaching multiple strategies on students' ability to solve problems correctly is known as operational knowledge, while awareness of mathematical concepts is known as theoretical knowledge.

A. True
B. False

Provide Instruction in Multiple Strategies

20. When teaching multiple strategies, it is beneficial to periodically employ unsuccessful strategies and demonstrate changing to an alternate strategy to show students that problems are not always solved easily the first time.

A. True
B. False

Ask Students to Generate and Share Multiple Strategies for Solving a Problem

21. When demonstrating the use of multiple strategies, teachers should randomly call on students to share their strategies, rather than selecting students purposefully based on the strategies they have used to solve the problem.

A. True
B. False

Potential Roadblocks and Solutions

22. In order to have enough time in their math class for students to present and discuss multiple strategies, teachers can:

A. Have students complete a problem-solving task at the beginning of the math class as a warm-up activity and devote 5-10 minutes to sharing and discussion
B. Purposefully select the two most effective strategies for sharing and discussing
C. Encourage students to share their strategies by writing them on the board for classmates to see
D. Have students take turns sharing their own strategies for 15-20 minutes at the beginning of class

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How to Carry out the Recommendation-Describe Relevant Mathematical Concepts and Notation

23. Teachers can turn problem-solving activities into learning opportunities by connecting students' intuitive understanding to formal mathematical concepts and notation, as students tend to enter school with ways of making sense of math that are:

A. Non-conventional and globally constructed
B. Informal and personally constructed
C. Ambiguous and individually constructed
D. Inaccurate and subjectively constructed

Help Students Make Sense of Algebraic Notation

24. Which of the following is NOT one of the expert recommendations for helping students understand symbolic notation used in algebra?

A. Provide familiar arithmetic problems as an intermediate step before asking students to translate a problem into an algebraic equation
B. Revisit students' earlier knowledge of simple arithmetic to help connect what they already know with new information
C. Have students explain each component of an algebraic equation by having them link the equation back to the problem they are solving
D. Introduce concepts at a quick pace in order to facilitate familiarity and increase comfort level

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25. Teachers should encourage students to use logical variables that relate to the items in the problem rather than arbitrary ones, as this will help clarify the abstract role that variables play in representing quantities.

A. True
B. False

Conclusion

26. When teachers state a prompt in front of the class and describe how they used it to solve a problem, students see how prompts or items from a task list are used and how people think as they solve problems.

A. True
B. False

Appendix D

27. Each of the following is an accurate statement about procedural knowledge and flexibility EXCEPT:

A. Procedural knowledge relates to whether students choose mathematical operations and procedures that will help them solve the problem and to how well they carry out the operations and procedures they choose to use
B. One way for students to express their procedural knowledge is to accurately and completely explain the operations and ideas used to solve a problem
C. Procedural flexibility relates to whether students can identify and carry out multiple methods to solve math problems
D. If students can adaptively choose the most appropriate strategy for a particular problem and can attempt to solve a math problem in multiple ways, then they have likely developed procedural flexibility

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Recommendation 2

28. Several studies, including some that also involved teacher modeling, prompted students to self-question or to complete tasks or steps while problem solving, and in all of these studies, the intervention's effects were positive.

A. True
B. False

Recommendation 3

29. While studies clearly indicated that students with disabilities performed better when specifically taught how to use different visual representations for different types of problem-solving, the results were inconclusive for general-education students.

A. True
B. False

Recommendation 5

30. Solving intermediate arithmetic problems before representing them with algebraic notation may help students understand problem structure using the mathematical knowledge they already possess, and students can then use this existing knowledge to more easily determine algebraic notation.

A. True
B. False
