TeachME PROFESSIONAL DEVELOPMENT
CEU's for Teachers and Educators

Teaching Math to Young Children

Levels of Evidence for Practice Guides

Institute of Education Sciences Levels of Evidence for Practice Guides

This section provides information about the role of evidence in Institute of Education Sciences' (IES) What Works Clearinghouse (WWC) practice guides. It describes how practice guide panels determine the level of evidence for each recommendation and explains the criteria for each of the three levels of evidence (strong evidence, moderate evidence, and minimal evidence).

The level of evidence assigned to each recommendation in this practice guide represents the panel's judgment of the quality of the existing research to support a claim that, when these practices were implemented in past research, favorable effects were observed on student outcomes. After careful review of the studies supporting each recommendation, panelists determine the level of evidence for each recommendation using the criteria in Table 1. The panel first considers the relevance of individual studies to the recommendation and then discusses the entire evidence base, taking the following into consideration:

- the number of studies
- the study designs
- the internal validity of the studies
- whether the studies represent the range of participants and settings on which the recommendation is focused
- whether findings from the studies can be attributed to the recommended practice
- whether findings in the studies are consistently positive

A rating of *strong evidence* refers to consistent evidence that the recommended strategies, programs, or practices improve student outcomes for a diverse population of students.[1] In other words, there is strong causal and generalizable evidence.

A rating of *moderate evidence* refers either to evidence from studies that allow strong causal conclusions but cannot be generalized with assurance to the population on which a recommendation is focused (perhaps because the findings have not been widely replicated) or to evidence from studies that are generalizable but have some causal ambiguity. It also might be that the studies that exist do not specifically examine the outcomes of interest in the practice guide, although they may be related.

A rating of *minimal evidence* suggests that the panel cannot point to a body of research that demonstrates the practice's positive effect on student achievement. In some cases, this simply means that the recommended practices would be difficult to study in a rigorous, experimental fashion;[2] in other cases, it means that researchers have not yet studied this practice, or that there is weak or conflicting evidence of effectiveness. A minimal evidence rating does not indicate that the recommendation is any less important than other recommendations with a strong or moderate evidence rating.

In developing the levels of evidence, the panel considers each of the criteria in Table 1. The level of evidence rating is determined by the lowest rating achieved for any individual criterion. Thus, for a recommendation to get a strong rating, the research must be rated as strong on each criterion. If at least one criterion receives a rating of moderate and none receive a rating of minimal, then the level of evidence is determined to be moderate. If one or more criteria receive a rating of minimal, then the level of evidence is determined to be minimal.

Table 1. Institute of Education Sciences levels of evidence for practice guides

Criteria	STRONG Evidence Base	MODERATE Evidence Base	MINIMAL Evidence Base
Validity	High internal validity (high-quality causal designs). Studies must meet WWC standards with or without reservations.[3] **AND** High external validity (requires multiple studies with high-quality causal designs that represent the population on which the recommendation is focused). Studies must meet WWC standards with or without reservations.	High internal validity but moderate external validity (i.e., studies that support strong causal conclusions but generalization is uncertain). **OR** High external validity but moderate internal validity (i.e., studies that support the generality of a relation but the causality is uncertain).[4]	The research may include evidence from studies that do not meet the criteria for moderate or strong evidence (e.g., case studies, qualitative research).
Effects on relevant outcomes	Consistent positive effects without contradictory evidence (i.e., no statistically significant negative effects) in studies with high internal validity.	A preponderance of evidence of positive effects. Contradictory evidence (i.e., statistically significant negative effects) must be discussed by the panel and considered with regard to relevance to the scope of the guide and intensity of the recommendation as a component of the intervention evaluated.	There may be weak or contradictory evidence of effects.
Relevance to scope	Direct relevance to scope (i.e., ecological validity)—relevant context (e.g., classroom vs. laboratory), sample (e.g., age and characteristics), and outcomes evaluated.	Relevance to scope (ecological validity) may vary, including relevant context (e.g., classroom vs. laboratory), sample (e.g., age and characteristics), and outcomes evaluated. At least some research is directly relevant to scope (but the research that is relevant to scope does not qualify as strong with respect to validity).	The research may be out of the scope of the practice guide.
Relationship between research and recommendations	Direct test of the recommendation in the studies or the recommendation is a major component of the intervention tested in the studies.	Intensity of the recommendation as a component of the interventions evaluated in the studies may vary.	Studies for which the intensity of the recommendation as a component of the interventions evaluated in the studies is low; and/or the recommendation reflects expert opinion based on reasonable extrapolations from research.

Table 1. Institute of Education Sciences levels of evidence for practice guides *(continued)*

Criteria	STRONG Evidence Base	MODERATE Evidence Base	MINIMAL Evidence Base
Panel confidence	Panel has a high degree of confidence that this practice is effective.	The panel determines that the research does not rise to the level of strong but is more compelling than a minimal level of evidence. Panel may not be confident about whether the research has effectively controlled for other explanations or whether the practice would be effective in most or all contexts.	In the panel's opinion, the recommendation must be addressed as part of the practice guide; however, the panel cannot point to a body of research that rises to the level of moderate or strong.
Role of expert opinion	Not applicable	Not applicable	Expert opinion based on defensible interpretations of theory (theories). (In some cases, this simply means that the recommended practices would be difficult to study in a rigorous, experimental fashion; in other cases, it means that researchers have not yet studied this practice.)
When assessment is the focus of the recommendation	For assessments, meets the standards of *The Standards for Educational and Psychological Testing*.[5]	For assessments, evidence of reliability that meets *The Standards for Educational and Psychological Testing* but with evidence of validity from samples not adequately representative of the population on which the recommendation is focused.	Not applicable

The panel relied on WWC evidence standards to assess the quality of evidence supporting educational programs and practices. The WWC evaluates evidence for the causal validity of instructional programs and practices according to WWC standards. Information about these standards is available at http://whatworks.ed.gov. Eligible studies that meet WWC evidence standards for group designs or meet evidence standards with reservations are indicated by **bold text** in the endnotes and references pages.

Introduction to the *Teaching Math to Young Children* Practice Guide

Children demonstrate an interest in math well before they enter school.[6] They notice basic geometric shapes, construct and extend simple patterns, and learn to count. The *Teaching Math to Young Children* practice guide presents five recommendations designed to capitalize on children's natural interest in math to make their preschool and school experience more engaging and beneficial. These recommendations are based on the panel members' expertise and experience and on a systematic review of the available literature. The first two recommendations identify which early math content areas[7] (number and operations, geometry, patterns, measurement, and data analysis)[8] should be a part of the preschool, prekindergarten, and kindergarten curricula, while the last three recommendations discuss strategies for incorporating this math content in classrooms. The recommendations in this guide can be implemented using a range of resources, including existing curricula.

In recent years, there has been an increased emphasis on developing and testing new early math curricula.[9] The development of these curricula was informed by research focused on the mechanisms of learning math,[10] and recent studies that test the impact of early math curricula show that devoting time to specific math activities as part of the school curriculum is effective in improving children's math learning before and at the beginning of elementary school.[11] Research evidence also suggests that children's math achievement when they enter kindergarten can predict later reading achievement; foundational skills in number and operations may set the stage for reading skills.[12]

Despite these recent efforts, many children in the United States lack the opportunity to develop the math skills they will need for future success. Research indicates that individual differences among children are evident before they reach school.[13] Children who begin with relatively low levels of math knowledge tend to progress more slowly in math and fall further behind.[14] In addition to these differences within the United States, differences in achievement between American children and students in other countries can be observed as early as the start of kindergarten.[15] Low achievement at such an early age puts U.S. children at a disadvantage for excelling in math in later years.[16] The panel believes that the math achievement of young children can be improved by placing more emphasis on math instruction throughout the school day.

This practice guide provides concrete suggestions for how to increase the emphasis on math instruction. It identifies the early math content areas that are important for young children's math development and suggests instructional techniques that can be used to teach them.

The panel's recommendations are in alignment with state and national efforts to identify what children should know, such as the Common Core State Standards (CCSS) and the joint position statement from the National Association for the Education of Young Children (NAEYC) and National Council of Teachers of Math (NCTM).[17] The early math content areas described in Recommendations 1 and 2 align with the content area objectives for kindergartners in the CCSS.[18] The panel recommends teaching these early math content areas using a developmental progression, which is consistent with the NAEYC/NCTM's recommendation to use curriculum based on known sequencing of mathematical ideas. Some states, such as New York, have adopted the CCSS and developed preschool standards that support the CCSS. *The New York State Foundation to the Common Core* is guided by principles that are similar to recommendations in this guide.[19]

The recommendations also align with the body of evidence in that the recommended practices are frequently components of curricula that are used in preschool, prekindergarten, and kindergarten classrooms. However, the practices are part of a larger curriculum, so their

effectiveness has not been examined individually. As a result, the body of evidence does not indicate whether each recommendation would be effective if implemented alone. However, the evidence demonstrates that when all of the recommendations are implemented together, students' math achievement improves.[20] Therefore, the panel suggests implementing all five recommendations in this guide together to support young children as they learn math. The first two recommendations identify important content areas. Recommendation 1 identifies number and operations as the primary early math content area, and Recommendation 2 describes the importance of teaching four other early math content areas: geometry, patterns, measurement, and data analysis. Recommendations 3 and 4 outline how teachers can build on young children's existing math knowledge, monitor progress to individualize instruction, and eventually connect children's everyday informal math knowledge to the formal symbols that will be used in later math instruction. Finally, Recommendation 5 provides suggestions for how teachers can dedicate time to math each day and link math to classroom activities throughout the day.

Scope of the practice guide

Audience and grade level. This guide is intended for the many individuals involved in the education of children ages 3 through 6 attending preschool, prekindergarten, and kindergarten programs. Teachers of young children may find the guide helpful in thinking about what and how to teach to prepare children for later math success. Administrators of preschool, prekindergarten, and kindergarten programs also may find this guide helpful as they prepare teachers to incorporate these early math content areas into their instruction and use the recommended practices in their classrooms. Curriculum developers may find the guide useful when developing interventions, and researchers may find opportunities to extend or explore variations in the body of evidence.

Common themes. This guide highlights three common themes for teaching math to young children.

- **Early math instruction should include multiple content areas.** Understanding the concept of number and operations helps create the foundation of young children's math understanding, and is the basis for Recommendation 1. Because there is much more to early math than understanding number and operations, the panel also reviewed the literature on instruction in geometry, patterns, measurement, and data analysis, as summarized in Recommendation 2. Giving young children experience in early math content areas other than number and operations helps prepare them for the different math subjects they will eventually encounter in school, such as algebra and statistics, and helps them view and understand their world mathematically.

- **Developmental progressions can help guide instruction and assessment.** The order in which skills and concepts build on one another as children develop knowledge is called a developmental progression. Both Recommendation 1 and Recommendation 2 outline how various early math content areas should be taught according to a developmental progression. There are different developmental progressions for each skill. These developmental progressions are important for educators to understand because they show the order in which young children typically learn math concepts and skills. The panel believes educators should pay attention to the order in which math instruction occurs and ensure that children are comfortable with earlier steps in the progression before being introduced to more complex steps. Understanding developmental progressions is also necessary to employ progress monitoring, a form of assessment that tracks individual children's success along the steps in the progression, as described in Recommendation 3.[21] The panel developed a specific developmental progression for

teaching number and operations based on their expertise and understanding of the research on how children learn math (see Table 3). The panel acknowledges that different developmental progressions exist; for example, the *Building Blocks* curriculum is based on learning trajectories that are similar but not identical to the developmental progression presented.[22] For a discussion of learning trajectories in mathematics broadly, as well as the connection between learning trajectories, instruction, assessment, and standards, see Daro, Mosher, and Corcoran (2011).

Developmental progressions refer to sequences of skills and concepts that children acquire as they build math knowledge.

- **Children should have regular and meaningful opportunities to learn and use math.** The panel believes that math should be a topic of discussion throughout the school day and across the curriculum. Early math instruction should build on children's current understanding and lay the foundation for the formal systems of math that will be taught later in school. These instructional methods guide Recommendations 4 and 5, which focus on embedding math instruction throughout the school day.[23]

Summary of the recommendations

Recommendation 1 establishes number and operations as a foundational content area for children's math learning. The recommendation presents strategies for teaching number and operations through a developmental progression. Teachers should provide opportunities for children to subitize small collections, practice counting, compare the magnitude of collections, and use numerals to quantify collections. Then, teachers should encourage children to solve simple arithmetic problems.

Recommendation 2 underscores the importance of teaching other early math content areas—specifically geometry, patterns, measurement, and data analysis—in preschool, prekindergarten, and kindergarten. The panel reiterates the importance of following a developmental progression to organize the presentation of material in each early math content area.

Recommendation 3 describes the use of progress monitoring to tailor instruction and build on what children know. The panel recommends that instruction include first determining children's current level of math knowledge based on a developmental progression and then using the information about children's skills to customize instruction. Monitoring children's progress throughout the year can then be an ongoing part of math instruction.

Recommendation 4 focuses on teaching children to view their world mathematically. The panel believes children should begin by using informal methods to represent math concepts and then learn to link those concepts to formal math vocabulary and symbols (such as the word *plus* and its symbol, +). Teachers can use open-ended questions and math conversation as a way of helping children to recognize math in everyday situations.

Recommendation 5 encourages teachers to set aside time each day for math instruction and to look for opportunities to incorporate math throughout the school day and across the curriculum.

Summary of supporting research

The panel used a substantial amount of national and international[24] research to develop this practice guide. This research was used to inform the panel's recommendations and to rate the level of evidence for the effectiveness of these recommendations. In examining the research base for practices and strategies for teaching math to young children, the panel paid particular attention to experimental and quasi-experimental studies that meet What Works Clearinghouse (WWC) standards.

The panel considered two bodies of literature to develop the recommendations in the practice guide: (1) theory-driven research, including developmental research[25] and (2) research on effective practice. The theory-driven research provided a foundation from which the panel developed recommendations by providing an understanding of how young children learn math. As this first body of literature did not examine the effectiveness of interventions, it was not reviewed under WWC standards, but it did inform the panel's expert opinion on how young children learn math. The second body of literature provided evidence of the effectiveness of practices as incorporated in existing interventions. This body of literature was eligible for review under WWC standards and, along with the panel's expert opinion, forms the basis for the levels of evidence assigned to the recommendations.

Recommendations were developed in an iterative process. The panel drafted initial recommendations that were based on its expert knowledge of the research on how young children learn math. The WWC then conducted a systematic review of literature following the protocol to identify and review the effectiveness literature relevant to teaching math to young children. The findings of the systematic review were then evaluated to determine whether the literature supported the initial recommendations or suggested other practices that could be incorporated in the recommendations. The final recommendations, which are presented in this guide, reflect the panel's expert opinion and interpretation of both bodies of literature.

The research base for this guide was identified through a comprehensive search for studies evaluating instructional practices for teaching math to children in preschool, prekindergarten, or kindergarten programs. The *Scope of the practice guide* section (p. 8) describes some of the criteria and themes used as parameters to help shape the literature search. A search for literature related to early math learning published between 1989 and 2011 yielded more than 2,300 citations. Of the initial set of studies, 79 studies used experimental and quasi-experimental designs to examine the effectiveness of the panel's recommendations. From this subset, 29 studies met WWC standards and were related to the panel's recommendations.[26]

The strength of the evidence for the five recommendations varies, and the level of evidence ratings are based on a combination of a review of the body of evidence and the panel's expertise. The supporting research provides a moderate level of evidence for Recommendation 1 and a minimal level of evidence for Recommendations 2–5. Although four recommendations were assigned a minimal level of evidence rating, all four are supported by studies with positive effects. These studies include a combination of practices that are covered in multiple recommendations; therefore, it was not possible to attribute the effectiveness of the practice to any individual recommendation.[27] For example, teaching the content area of number and operations, along with other math content areas like geometry, patterns, and data analysis, was often a common component of effective comprehensive curricula. Additionally, while the panel suggests that teachers assess children's understanding on a regular basis and use that information to tailor instruction, the panel could not find research that isolated the impact of progress monitoring on children's math knowledge. Similarly, there is limited evidence on the effectiveness of teaching children to view and describe their world mathematically, as this component was never separated from other aspects of the intervention. Finally, there also is limited evidence on the effectiveness of time spent on math because there is a lack of research in which the only difference between groups was instructional time for math.

Although the research base does not provide direct evidence for all recommendations in isolation, the panel believes the recommendations in this guide are necessary components of early math instruction based on panel

members' knowledge of and experience working in preschool, prekindergarten, and kindergarten classrooms. The panel identified evidence indicating that student performance improves when these recommendations are implemented together.

Table 2 shows each recommendation and the level of evidence rating for each one as determined by the panel. Following the recommendations and suggestions for carrying out the recommendations, Appendix D presents more information on the body of evidence supporting each recommendation.

Table 2. Recommendations and corresponding levels of evidence

	Levels of Evidence		
Recommendation	**Strong Evidence**	**Moderate Evidence**	**Minimal Evidence**
1. Teach number and operations using a developmental progression.		◆	
2. Teach geometry, patterns, measurement, and data analysis using a developmental progression.			◆
3. Use progress monitoring to ensure that math instruction builds on what each child knows.			◆
4. Teach children to view and describe their world mathematically.			◆
5. Dedicate time each day to teaching math, and integrate math instruction throughout the school day.			◆

Teach number and operations using a developmental progression.

Early experience with number and operations is fundamental for acquiring more complex math concepts and skills.[28] In this recommendation, the panel describes the main aspects of early number knowledge, moving from basic number skills to operations.

Effective instruction depends on identifying the knowledge children already possess and building on that knowledge to help them take the next developmental step. Developmental progressions can help identify the next step by providing teachers with a road map for developmentally appropriate instruction for learning different skills.[29] For example, teachers can use progressions to determine the developmental prerequisites for a particular skill and, if a child achieves the skill, to help determine what to teach next. Similarly, when a child is unable to a grasp a concept, developmental prerequisites can inform a teacher what skills a child needs to work on to move forward. In other words, developmental progressions can be helpful aids when tailoring instruction to individual needs, particularly when used in a deliberate progress monitoring process (see Recommendation 3). Although there are multiple developmental progressions that may vary in their focus and exact ordering,[30] the steps in this recommendation follow a sequence that the panel believes represents core areas of number knowledge (see Table 3).[31] Additional examples of developmental progressions may be found in early math curricula, assessments, and research articles.

With each step in a developmental progression, children should first focus on working with small collections of objects (one to three items) and then move to progressively larger collections of objects. Children may start a new step with small numbers before moving to larger numbers with the previous step.[32]

Table 3. Examples of a specific developmental progression for number knowledge

Developmental Progression

Subitizing (small-number recognition)	Subitizing refers to a child's ability to immediately recognize the total number of items in a collection and label it with an appropriate number word. When children are presented with many different examples of a quantity (e.g., two eyes, two hands, two socks, two shoes, two cars) labeled with the same number word, as well as non-examples labeled with other number words (e.g., three cars), children construct precise concepts of one, two, and three.
	A child is ready for the next step when, for example, he or she is able to see one, two, or three stickers and immediately—without counting—state the correct number of stickers.
Meaningful object counting	Meaningful object counting is counting in a one-to-one fashion and recognizing that the last word used while counting is the same as the total (this is called the cardinality principle).
	A child is ready for the next step when, for example, if given five blocks and asked, "How many?" he or she counts by pointing and assigning one number to each block: "One, two, three, four, five," and recognizes that the total is "five."
Counting-based comparisons of collections larger than three	Once children can use small-number recognition to compare small collections, they can use meaningful object counting to determine the larger of two collections (e.g., "seven" items is more than "six" items because you have to count further).
	A child is ready for the next step when he or she is shown two different collections (e.g., nine bears and six bears) and can count to determine which is the larger one (e.g., "nine" bears is more).
Number-after knowledge	Familiarity with the counting sequence enables a child to have number-after knowledge—i.e., to enter the sequence at any point and specify the next number instead of always counting from one.
	A child is ready for the next step when he or she can answer questions such as, "What comes after five?" by stating "five, six" or simply "six" instead of, say, counting "one, two, … six."
Mental comparisons of close or neighboring numbers	Once children recognize that counting can be used to compare collections and have number-after knowledge, they can efficiently and mentally determine the larger of two adjacent or close numbers (e.g., that "nine" is larger than "eight").
	A child has this knowledge when he or she can answer questions such as, "Which is more, seven or eight?" and can make comparisons of other close numbers.
Number-after equals one more	Once children can mentally compare numbers and see that "two" is one more than "one" and that "three" is one more than "two," they can conclude that any number in the counting sequence is exactly one more than the previous number.
	A child is ready for the next step when he or she recognizes, for example, that "eight" is one more than "seven."

Summary of evidence: Moderate Evidence

The panel assigned a rating of *moderate evidence* to this recommendation based on their expertise and 21 randomized controlled trials[33] and 2 quasi-experimental studies[34] that met WWC standards and examined interventions that included targeted instruction in number and operations. The studies supporting this recommendation were conducted in preschool, prekindergarten, and kindergarten classrooms.

The research shows a strong pattern of positive effects on children's early math achievement across a range of curricula with a focus on number and operations. Eleven studies evaluated the effectiveness of instruction in only number and operations, and all 11 studies found at least one positive effect on basic number concepts or operations.[35] The other 12 studies evaluated the effectiveness of instruction in number and operations in the context of broader curricula.

None of the 23 studies that contributed to the body of evidence for Recommendation 1 evaluated the effectiveness of instruction based on a developmental progression compared to instruction that was not guided by a developmental progression. As a result, the panel could not identify evidence for teaching based on any particular developmental progression. Additional research is needed to identify the developmental progression that reflects how most children learn math. Yet based on their expertise, and the pattern of positive effects for interventions guided by a developmental progression, the panel recommends the use of a developmental progression to guide instruction in number and operations.[36]

Positive effects were found even in studies in which the comparison group also received instruction in number and operations.[37] The panel classified an intervention as having a focus on number and operations if it included instruction in at least one concept related to number and operations. The panel found that the math instruction received by the comparison group differed across the studies, and in some cases the panel was unable to determine what math instruction the comparison group received.[38] Despite these limitations, the panel believes interventions with a focus on number and operations improve the math skills of young children.

Although the research tended to show positive effects, some of these effects may have been driven by factors other than the instruction that was delivered in the area of number and operations. For example, most interventions included practices associated with multiple recommendations in this guide (also known as multi-component interventions).[39] As a result, it was not possible to determine whether findings were due to a single practice—and if so, which one—or a combination of practices that could be related to multiple recommendations in this guide. While the panel cannot determine whether a single practice or combination of practices is responsible for the positive effects seen, the pattern of positive effects indicates instruction in teaching number and operations will improve children's math skills.

The panel identified five suggestions for how to carry out this recommendation.

How to carry out the recommendation

1. First, provide opportunities for children to practice recognizing the total number of objects in small collections (one to three items) and labeling them with a number word without needing to count them.

Being able to correctly determine the number of objects in a small collection is a critical skill that children must develop to help them learn more complex skills, including counting larger collections and eventually adding and subtracting. To give children experience with subitizing[40] (also known as small-number recognition), teachers should ask children to answer the question "How many (name of object) do you see?" when looking at collections of one to three objects.[41] As described in the first step of Table 3, children should practice stating the total for small collections without necessarily counting. Research indicates that young children can learn to use subitizing to successfully determine the quantity of a collection.[42]

Transitions between classroom activities can provide quick opportunities for children to practice subitizing. Teachers can find collections of two or three of the same object around the classroom (e.g., fingers, unit cubes, seashells, chips). Teachers can ask "How many ____?" (without counting) before transitioning to the next activity. Another way to help children practice immediately recognizing quantities is during snack time, when, for example, a teacher can give a child two crackers and then ask the child how many crackers he or she has. Practicing subitizing in meaningful, everyday contexts such as snack time, book reading, and other activities can reinforce children's math skills.

Children can also practice subitizing while working in small groups. The *Basic Hiding* game is one example of a subitizing activity that can be used with small groups of children (see Example 1).

Once children have some experience recognizing and labeling small collections of similar objects (e.g., three yellow cubes), teachers can introduce physically dissimilar items of the same type (e.g., a yellow cube, a green cube, and a red cube). Eventually, teachers can group unrelated items (e.g., a yellow cube, a toy frog, and a toy car) together and ask children, "How many?" Emphasizing that collections of three similar objects and three dissimilar objects are both "three" will help children construct a more abstract or general concept of number.[44]

As children begin to learn these concepts, they may overgeneralize. Early development is often marked by the overgeneralization of terms (e.g., saying "two" and then "three" or another number such as "five" to indicate "many").[45] The panel believes one way to help children define the limits of a number concept is to contrast examples of a number with non-examples. For instance, in addition to labeling three toys as "three," labeling four toys as "not three" (e.g., "That's four toys, not three toys") can help children clearly understand the meaning of "three." Once children are accustomed to hearing adults labeling examples and non-examples, teachers can have children find their own examples and non-examples (e.g., "Can someone find two toys? Now, what is something that is *not* two?").[46]

Example 1. The *Basic Hiding* game[43]

Objective

Practice subitizing—immediately recognizing and labeling small numbers and constructing a concept of one to three—and the concept of number constancy (rearranging items in a set does not change its total).

Materials needed:

- Objects. Use a small set of identical objects early on and later advance to larger sets or sets of similar, but not identical, objects.
- Box, cloth, or other item that can be used to hide the objects.

Directions: With a small group of children, present one to three objects on a mat for a few seconds. Cover them with a cloth or box and then ask the children, "Who can tell me how many (name of objects) I am hiding?" After the children have answered, uncover the objects so that the objects can be seen. The children can count to check their answer, or the teacher can reinforce the answer by saying, for example, "Yes, two. See, there are two (objects) on the mat: one, two." Continue the game with different numbers of objects arranged in different ways. Teachers can also tailor the *Basic Hiding* game for use with the whole class or individual children.

Early math content areas covered

- Subitizing
- Increasing magnitude up to five items

Monitoring children's progress and tailoring the activity appropriately

- Vary the number of objects to determine whether children are ready to use larger sets.
- If a child has difficulty, before covering the objects, ask the child how many items he or she sees. Then, cover the objects and ask again. For larger collections (greater than three), allow the child to check his or her answer by counting.

Integrating the activity into other parts of the day

- Consider playing the game at various points during the day with different sets of objects, including objects that are a part of children's everyday experience (e.g., spoons and blocks).

Using the activity to increase math talk in the classroom

- Use both informal ("more" or "less") and formal ("add" and "subtract") language to describe changing the number of objects in the set.

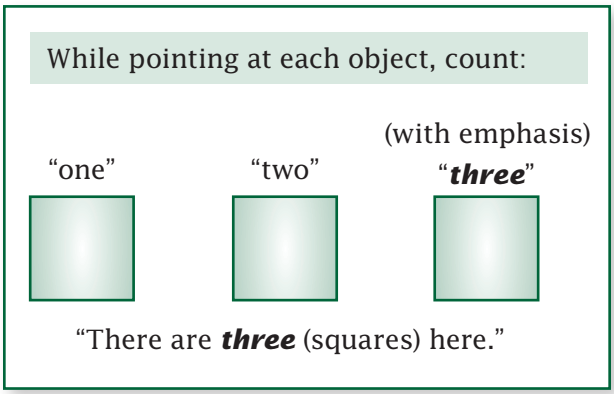
2. Next, promote accurate one-to-one counting as a means of identifying the total number of items in a collection.

Small-number recognition provides a basis for learning the one-to-one counting principle in a meaningful manner.[47] Often, children begin learning about number from an early age by reciting the count sequence ("one, two, three, four…"). But learning to assign the numbers of the count sequence to a collection of objects that are being counted can be a challenging step. Once children are able to reliably recognize and label collections of one to three items immediately (without counting), they have started to connect numbers with quantity. As illustrated in the second step of Table 3, they should then begin to use one-to-one counting to identify "how many" are in larger collections.[48]

To count accurately, one—and only one—number word must be assigned to each item in the collection being counted. For example, when counting four pennies, children must point to a penny and say "one," point to a second penny and say "two," point to a third penny and say "three," and point to the final penny and say "four." During this activity the child will need to keep track of which pennies have been labeled and which still need to be labeled. The child can also practice recognition of the cardinality principle: that the last number word is the total (cardinal value) of the collection. Although children can learn to count one-to-one by rote, they typically do not recognize at the outset that the goal of this skill is to specify the total of a collection or how many there are. For example, when asked how many they just counted, some children count again or just guess. By learning one-to-one counting with small collections that they already recognize, children can see that the last word used in the counting process is the same as the total.[49]

Teachers should model one-to-one counting with one to three items—collections children can readily recognize and label—and emphasize or repeat the last number word used in the counting process, as portrayed in Figure 1.[50] By practicing with small collections they can already recognize, preschool, prekindergarten, and kindergarten children will begin to learn that counting is a method for answering the question, "How many?"[51]

Figure 1. Modeling one-to-one counting with one to three items



Once children can find the total with small collections, they are ready to count larger collections (four to ten objects). For example, by counting seven objects one by one ("one, two, three, four, five, six, and seven"), the child figures out that "seven" is the total number of objects in the set. Teachers can also challenge children by having them count sounds (e.g., clapping a certain number of times and asking, "How many claps?") or actions (e.g., counting the number of hops while hopping on one foot).

Children can use everyday situations and games, such as *Hidden Stars* (see Example 2), to practice counting objects and using the last number counted to determine the total quantity. This game is similar to the *Basic Hiding* game; however, in *Hidden Stars*, the goal is to count the objects first and then use that number to determine the total quantity (without recounting). It is important to demonstrate that counting is not dependent upon the order of the objects. That is, children can start from the front of a line of blocks or from the back of a line of blocks, and as long as they use one-to-one counting, they will get the same quantity.

Example 2. The *Hidden Stars* game[52]

Objective

Practice using one-to-one counting and the final number counted to identify "how many" objects.

Materials needed:

- Star stickers in varying quantities from one to ten, glued to 5-by-8-inch cards
- Paper for covering cards

Directions: Teachers can tailor the *Hidden Stars* game for use with the whole class, a small group, or individual children. Show children a collection of stars on an index card. Have one child count the stars. Once the child has counted the stars correctly, cover the stars and ask, "How many stars am I hiding?"

Early math content areas covered

- Counting
- Cardinality (using the last number counted to identify the total in the set)

Monitoring children's progress and tailoring the activity appropriately

- Work with children in a small group, noting each child's ability to count the stars with accuracy and say the amount using the cardinality principle (the last number counted represents the total).
- When children repeat the full count sequence, model the cardinality principle. For example, for four items, if a child repeats the count sequence, say, "One, two, three, *four*. So I need to remember *four*. There are four stars hiding."
- Have a child hide the stars while telling him or her how many there are, emphasizing the last number as the significant number.

Using the activity to increase math talk in the classroom

- Ask, "How many?" (e.g., "How many blocks did you use to build your house? How many children completed the puzzle?")

Errors in counting. When children are still developing counting skills, they will often make errors. Some errors are predictable. For example, some children will point to the same object more than once or count twice while pointing at only one object. Table 4 describes common counting errors and provides suggestions teachers can use to correct those errors when working with children in one-on-one or small-group situations.[53]

Table 4. Common counting errors

Type of Counting Error	Example	Remedy
SEQUENCE ERROR		
Saying the number sequence out of order, skipping numbers, or using the same number more than once.	"1 2 3 6 10" 	Practice reciting (or singing) the single-digit sequence, first focusing on one to ten, then later moving on to numbers greater than ten.
Struggling with the count sequence past twelve.	Skips 15: "1…13, 14, 16, 17, 18." Uses incorrect words: "1…13, 14, fiveteen." "1…18, 19, 10-teen" or "1…29, 20-ten, 20-eleven." Stops at a certain number: "1…20" (stops) "1…20" (starts from 1 again)	Highlight and practice exceptions, such as *fif + teen*. Fifteen and thirteen are commonly skipped because they are irregular. Recognize that a nine signals the end of a series and that a new one needs to begin (e.g., nineteen marks the end of the teens). Recognize that each new series (decade) involves combining a decade and the single-digit sequence, such as twenty, twenty plus one, twenty plus two, etc. Recognize the decade term that begins each new series (e.g., twenty follows nineteen, thirty follows twenty-nine, and so forth). This involves both memorizing terms such as ten, twenty, and thirty by rote and recognizing a pattern: "add *-ty* to the single-digit sequence" (e.g., *six + ty*, *seven + ty*, *eight + ty*, *nine + ty*).
COORDINATION ERROR		
Labeling an object with more than one number word.	"1 2 3 4 5,6" 	Encourage the child to slow down and count carefully. Underscore that each item needs to be tagged only once with each number word.
Pointing to an object but not counting it.	 "1 2 3 4"	Same as above.
KEEPING TRACK ERROR		
Recounting an item counted earlier.	"1 2 3 4 5 6" 	Help the child devise strategies for sorting counted items from uncounted items. For movable objects, for instance, have the child place counted items aside in a pile clearly separated from uncounted items. For pictured objects, have him or her cross off items as counted.
SKIM		
No effort at one-to-one counting or keeping track.	Waves finger over the collection like a wand (or jabs randomly at the collection) while citing the counting sequence (e.g., "1, 2, 3…9, 10").	Underscore that each item needs to be tagged with one and only one number word and help the child to learn processes for keeping track. Model the counting.
NO CARDINALITY RULE		
Not recognizing that the last number word used in the counting process indicates the total.	Asked how many, the child tries to recount the collection or simply guesses.	Play *Hidden Stars* with small collections of one to three items first and then somewhat larger collections of items.

3. Once children can recognize or count collections, provide opportunities for children to use number words and counting to compare quantities.

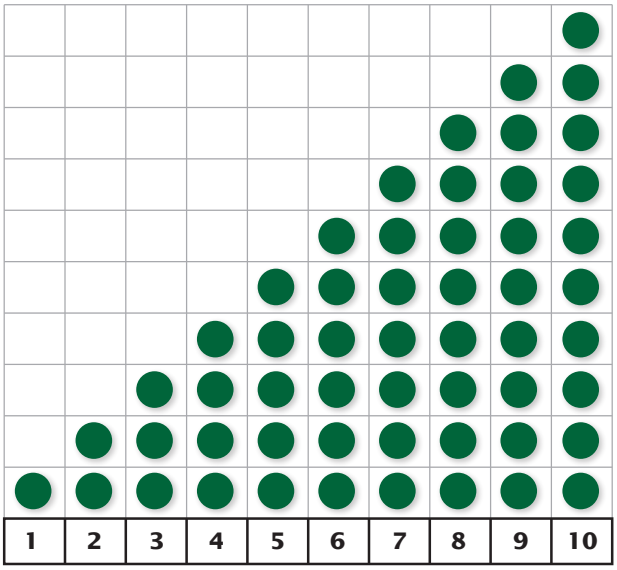
Once children can reliably determine how many objects are in a collection, either by subitizing or counting, teachers can provide them with opportunities to compare the magnitudes of different collections using number words (steps 3 through 6 in the developmental progression illustrated in Table 3).

To prepare children for making meaningful, verbal comparisons of magnitudes, teachers should ensure that they understand relational terms such as "more" and "fewer."[54] For example, a teacher can present two plates with obviously different numbers of cookies and ask, "Which plate has more cookies?" Teachers can also provide children with examples of "equal" by showing two groups with the same quantity of objects. Using these words provides children with the vocabulary for comparing larger collections.

Once children are comfortable making verbal comparisons, teachers should encourage them to use counting to compare the magnitudes of two collections.[55] Teachers can demonstrate that number words further along in the counting sequence represent larger collections.[56] Described in the third step of the developmental progression illustrated in Table 3, this is also known as the "increasing magnitude principle." A cardinality chart, as shown in Figure 2, visually underscores this principle and can be a useful tool to help children make number comparisons. Teachers can use the cardinality chart to demonstrate that the next number in the counting sequence is exactly one more than the previous number. Children can also use cardinality charts to reinforce the concepts of number-after relations, mental comparison of neighboring numbers, and the increasing magnitude principle.

Teachers can provide opportunities for practicing the application of the increasing magnitude principle while playing games that involve keeping score. A teacher can have two children compare their scores (represented by two sets of blocks or other markers) and see who won by counting. The teacher could summarize the process by saying, for instance, "Manny has five, but Keisha has one, two, three, four, five, *six*. Six is more than five, because six comes after five when we count."

Figure 2. Sample cardinality chart[57]



To prepare children to mentally compare numbers, teachers can help them master number-after relations (the fourth step in the developmental progression illustrated in Table 3). Everyday situations provide numerous opportunities to incorporate the use of number-after skills. For example, a teacher can say, "Jahael is having a birthday tomorrow; if Jahael is 4 now, how old will he be tomorrow?" or "We just passed Rooms 3 and 4. The next room should be what number?" or "Today is December 4. Tomorrow will be December what?"

Once children have mastered making concrete comparisons using one-to-one object counting and number-after relations, teachers can help them mentally compare neighboring number words (the fifth step in the developmental progression illustrated in Table 3). Teachers may

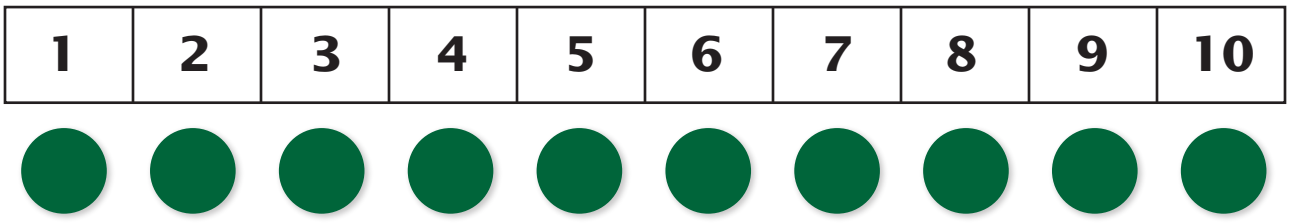
Some children may initially have trouble answering the question "What comes after six?" However, they may be successful if given a running start—counting from "one" up to a number (e.g., "What comes after 'One, two, three, four, five, six'?"). As children master number-after relations, they learn to determine the number after a counting word without using a running start.

find that a number list, or a series of numerals in order, can be used to compare numbers (see Figure 3).[58] Children can see which numbers are "more" or "fewer" based on the numbers' positions on the list. Number lists may be particularly helpful for comparing two collections: by counting with a number list, children can see that numbers earlier and later in the list denote lesser and greater cardinalities and, therefore, indicate smaller and larger quantities. As children practice, these comparisons can be done without the aid of a number list. Transitioning between activities provides a good opportunity to reinforce these types of questions. Children can answer a quick "Which is more?" question before transitioning to the next activity.

As children master the increasing magnitude principle and become comfortable with number-after relations, teachers can demonstrate that a number immediately after another is one more than its predecessor. Children may know, for example, that seven comes after six when we count and that seven is more than six, but they may not realize that seven is exactly one more than six and that each number in the counting sequence is exactly one more than the number before it.

A **number list** is a series of numerals beginning with 1 and ordered by magnitude. Number lists are similar to number lines; however, they do not include 0 and are an easier tool for young children to use when counting and learning numerals.

Figure 3. Sample number list



4. Encourage children to label collections with number words and numerals.

Once children have practiced recognizing, counting, and comparing quantities, teachers can introduce numerals to children as a way to represent a quantity.[59] Sometimes, children may begin to recognize the numerals in the world around them (e.g., on electronic devices, on street signs, or on television) before they are able to count. However, once children have a foundation for understanding number and counting, it may become easier for them to learn about numerals. Teachers can pair numerals with collections of objects around the classroom so that children start to learn, for example, that the numeral 3, three objects, and the spoken word "three" represent the same thing. If teachers use activity centers in their classrooms, they can number those centers with signs that have a numeral, dots representing the numeral, and the number word (e.g., "3, ▪ ▪ ▪, three"). Children who do not yet recognize numerals can use the dots to count and figure out what the numeral indicates. A wide variety of games, such as the memory game *Concentration: Numerals and Dots* (see Example 3), can serve as practice in identifying and reading numerals.

Example 3. The *Concentration: Numerals and Dots* game

Objective

Match numerals with corresponding quantities.

Materials needed:

- One set of twenty cards: ten cards with numerals from 1 to 10 along with the corresponding number of dots, and ten cards with pictures of objects (the numbers of objects corresponding to a numeral 1–10).
- For even more advanced play, once children are proficient at numerals 1–10, teachers can create cards for numerals 11–20.

Directions: Half of the cards have a numeral and dots to represent the amount (e.g., the numeral 3 and three dots) on one side, and the other half have pictures of collections of objects on one side (e.g., three horses, four ducks). The other side of each card is blank. The cards are placed face down, with the numeral cards in one area and the picture cards in another. A player chooses one numeral card and one picture card. If they match, then the player keeps those cards. Play continues until no further matching cards remain. The player with the most cards wins the game.

Early math content areas covered

- Numeral recognition.
- Corresponding quantity.
- If the objects in the pictures on the cards are in different orders, it can help reinforce the idea that appearance does not matter when it comes to number.

Monitoring children's progress and tailoring the activity appropriately

- Play the game with a small group of children, noting each child's progress in practicing and achieving the objectives.
- This game can be played with children who are not familiar with numeracy concepts. Use fewer cards, lower numbers, or cards with dots to scaffold. As children gain proficiency with the concepts, increase the number of cards and the size of the numbers.

Using the activity to increase math talk in the classroom

- Before asking, "How many?" ask, "How can we find out how many?"

5. Once children develop these fundamental number skills, encourage them to solve basic problems.

Using their number knowledge to solve arithmetic problems can give children a context to apply and expand this knowledge and gain confidence in their math ability.[60] Once children can determine the total number of items in a collection by using small-number recognition or counting and can understand the concepts of "more" and "fewer," they can explore the effects of adding and subtracting items from a collection. One way to help children apply their knowledge is to create activities that involve manipulating small

sets of objects.[61] Children can change small collections of objects by combining or removing objects (e.g., adding two blocks to three blocks) and then count to determine "how many" they have in the new collection. As children become more adept, teachers should present more difficult problems with slightly larger numbers. Problem solving can be useful even if children have not completely mastered fundamental number skills, as problem solving may serve as a vehicle for children's learning. Problem solving challenges children to use their math knowledge to answer and explain math-related questions, providing them with an opportunity to strengthen their math skills.

Teachers can use problem-solving tasks across classroom situations so children can see how to apply counting to solve everyday challenges. For example, when children are preparing to play games in small groups, the teacher can ask them to count how many groups there are and use that number to determine how many games to distribute. Once children can consistently use counting to solve simple problems, teachers can ask the class to help find out how many children are in attendance by first asking how many boys there are, then how many girls, and finally how many children in total. Examples with a real-life application for the skill (such as finding out how many children need a snack) are the most helpful to children's learning.[62]

Once children have experience with combining or separating objects in a collection they can see, they can do the same with collections of objects (e.g., pennies) when the final outcome is hidden from view.[63] This arrangement can be in a hiding game that is an extension of the *Basic Hiding* game (see Example 1) or *Hidden Stars* (see Example 2). Teachers can place three or four objects in a line while the children watch. Teachers can then cover the objects (with a cloth or with a box that has an opening on the side) and, while the objects are covered, take one or two additional objects and add them to the objects under the cover. (Alternatively, they can reach beneath the cover to take one or two objects away.) The children see the initial group of objects and the objects being added or taken away, but they do not see the final set of objects. The children must then determine, without looking at the final set of objects, how many are hiding. Children may solve this problem by counting on their fingers or in their heads. After the children give their answer, the teacher can take the cover away, and the children can count to check the answer.

Snack time is also a great opportunity to provide children with authentic comparisons of adding and subtracting or "more" and "fewer." As children receive or eat their snacks, they can count how many items they have. Teachers can also adapt this activity for children of varying skill levels by asking each child different questions, such as "How many will you have after you eat one?" or "How many will you have after your friend gives you one?" Because the number will change, this activity provides good practice for understanding comparisons of more and fewer and combining or removing objects.

Potential roadblocks and solutions

Roadblock 1.1. *I want to provide strong math foundations for my children, but I am not really comfortable with math myself.*

Suggested Approach. Teachers who are not comfortable teaching math can begin by looking for opportunities to teach math in regular activities or familiar situations. They can then design classroom projects that highlight the everyday uses of math. For example, quick counting tasks such as figuring out how many children need a snack, or how many mittens or hats children have, are easy ways to incorporate counting into everyday events. Activities such as setting up a pretend grocery store in the classroom allow children to practice counting food and money. Other examples include community

service projects, such as canned-food drives, which can provide opportunities for children to count, sort, label, and organize donations. Sports can also provide children with chances to practice math—for example, measuring the distance for a race on the playground, recording times, and making a chart to display results. Teachers can also consider sharing their own interests with children and highlighting whatever math is involved, such as the measurement involved in cooking or sewing, the geometry involved in woodworking, and so on.

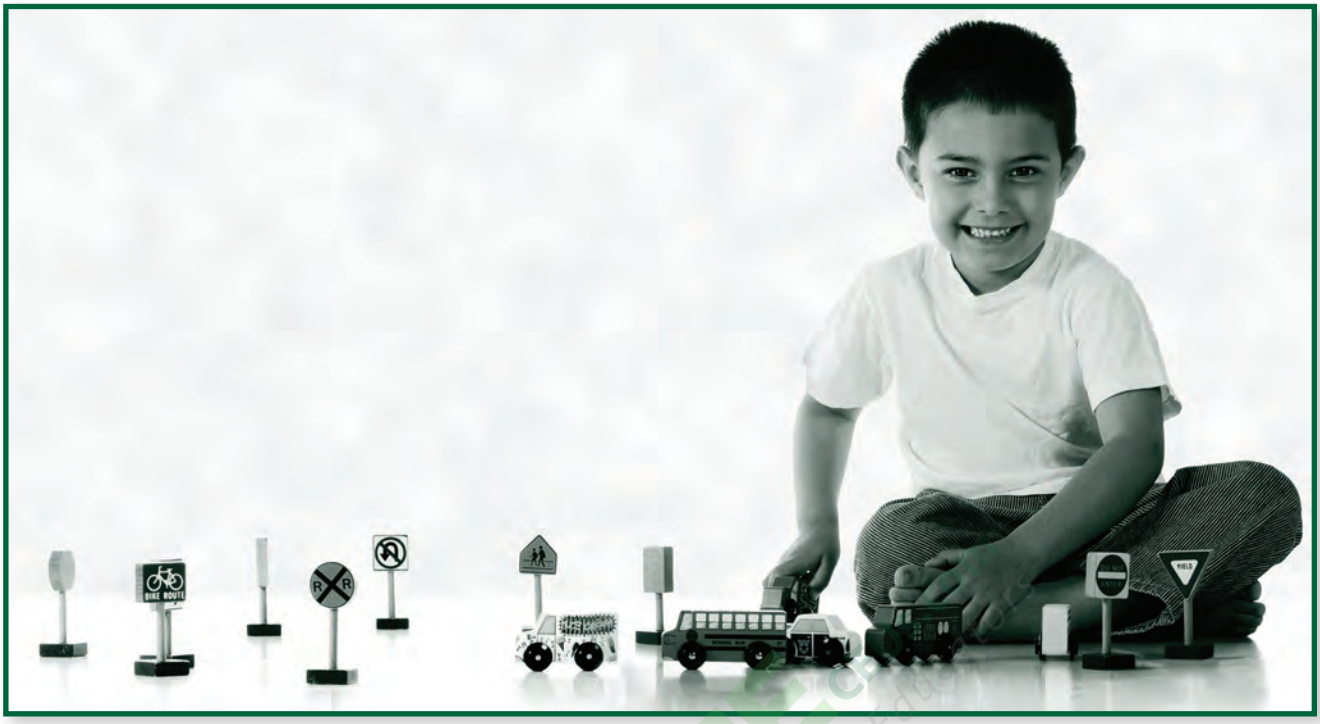
Roadblock 1.2. *Each child in the class is at a different level in the developmental progression I am using to guide instruction.*

Suggested Approach. Teachers can prepare whole-group lessons that target specific concepts and then use small-group activities in which children are grouped with peers who are at a similar level. One group of children can work on activities that are related to a more basic skill (such as counting objects), and another group of children can work on a more advanced activity (such as combining sets of objects and figuring out how many there are in total). Decreasing and increasing the quantity of a collection, using a color-coded die or dice labeled with numerals for playing board games, and increasing complexity of pattern activities while using the same objects are all simple ways to tailor activities. Alternatively, children can be grouped with other children who are at a more proficient level (heterogeneous groups) and can model the skill.

Roadblock 1.3. *A child is stuck at a particular point in the developmental progression.*

Suggested Approach. It may be useful to go back and make sure the child has learned the prerequisites for each step in the progression. Teachers can go back a step and give the child a chance to practice and reinforce skills in a previous level before trying the more challenging level again. It is also important to take into account what concept a child is developmentally ready to learn. Some children may need more practice with a particular skill before moving on to a more complex skill.

Recommendation 2



Teach geometry, patterns, measurement, and data analysis using a developmental progression.

Children's exposure to math should extend beyond number and operations to include a range of math content areas, including geometry (shapes and space), patterns, measurement, and data analysis.[64] As with Recommendation 1, these math content areas should be taught according to developmental progressions. Learning skills beyond number and operations creates a foundation for future math instruction, and children with strong backgrounds in these areas are more likely to succeed in later grades.[65] For example, early instruction in shapes and measurement lays the groundwork for future learning in geometry, and simple graphing exercises are the foundation for more advanced concepts such as statistics.

When children's understanding extends across a range of math content areas, they have the tools they need to explore and explain their world.[66] They learn that math is everywhere. Geometry is a part of their environment in the form of traffic signs, maps, and buildings. Patterns occur in nature. Measurements help children compare and quantify the things they experience. Collecting and organizing information, such as creating charts to display favorite animals or foods, allows children to find out more about one another.

The steps of this recommendation describe general developmental progressions through the early math content areas of geometry, patterns, measurement, and data analysis. Each component of this recommendation will indicate where to begin within each early math content area and how to progress to more advanced concepts.[67]

Summary of evidence: Minimal Evidence

The panel assigned a rating of *minimal evidence* to this recommendation based on their expertise and 12 randomized controlled trials[68] and 1 quasi-experimental study[69] that met WWC standards and examined interventions that provided targeted instruction in one or more of the early math content areas of Recommendation 2. The studies supporting this recommendation were conducted in preschool, prekindergarten, and kindergarten classrooms.

The 13 studies examined interventions that included different combinations of the early math content areas that are the focus of Recommendation 2.

- Ten separate interventions taught young children about geometry.[70] Each of these interventions was tested in at least one of the 12 studies. Positive effects were found for geometry, operations, and general numeracy outcomes, whether the teaching of geometry was part of a broader curriculum or the only component of the intervention. The interventions that taught geometry ranged from early math curricula with multiple units and lessons that focused on geometry,[71] to a curriculum with eight sessions in a four-week period (in addition to regular classroom instruction) that used a story to teach part-whole relations skills.[72]

- Eight interventions taught patterns.[73] These interventions were examined in 10 studies.[74] Six studies reported positive effects in the domains of general numeracy and geometry.[75] One study found positive effects in basic number concepts, operations, and patterns and classification.[76] One study found no discernible effects in operations, and two studies found no discernible effects in operations, general numeracy, and geometry.[77]

- Seven interventions taught measurement.[78] These interventions were examined in nine studies. Positive effects were found in the domains of general numeracy, geometry, and basic number concepts.[79]

- Six interventions taught data analysis.[80] These interventions were examined in eight studies. Six of the studies reported positive effects in the domains of general numeracy and basic number concepts.[81] The remaining two studies reported no discernible effects in the domains of operations, general numeracy, and geometry.[82]

The body of evidence assessed in relation to Recommendation 2 was promising. However, three issues with the evidence prevented the panel from assigning a moderate evidence rating to this recommendation.

First, none of the 13 studies that contributed to the body of evidence for Recommendation 2 evaluated the effectiveness of instruction based on a developmental progression compared to instruction that was not guided by a developmental progression. As a result, the panel could not identify evidence for teaching based on any particular developmental progression. Second, although the research tended to show positive effects, some of these effects may have been driven by factors other than the instruction that was delivered in the four content areas covered by Recommendation 2 and operations. For example, most interventions included practices associated with multiple recommendations in this guide (also known as multi-component interventions).[83] The panel was also concerned about the lack of specific information about how much time was spent on each early math content area in the intervention and comparison groups. Finally, many studies reported on outcomes that were not directly aligned with the early math content areas included in this recommendation.

Together, these three limitations resulted in the panel not being able to claim with certainty that the effects seen were due solely to targeted instruction in the early math content areas of geometry, patterns, measurement, and data analysis. Nevertheless, the panel believes the positive effects found for interventions based on a developmental progression when compared to instruction that does not appear to be based on a developmental progression support their recommendation to use a developmental progression to guide instruction. When combined with the positive effects found for interventions that included targeted instruction in geometry, patterns, measurement, and data analysis, the panel believes the studies generally support this recommendation, despite the limitations to the body of evidence.

The panel identified four suggestions for how to carry out this recommendation.

How to carry out the recommendation

1. Help children recognize, name, and compare shapes, and then teach them to combine and separate shapes.

Teachers should encourage children to recognize and identify shapes in their surrounding environment.[84] Children may find shapes in their drawings, bring an object from home that illustrates a particular shape, or locate shapes in the classroom.

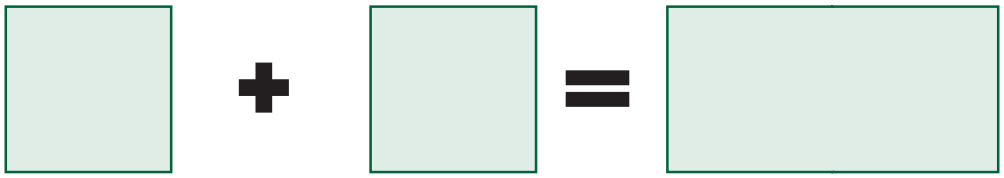
When children can confidently recognize shapes, teachers should then provide opportunities for children to name the critical attributes of shapes using standard geometric terms. A critical attribute of a shape is a characteristic shared by all examples of that shape. For example, all rectangles have four sides, and the opposite sides are equal and parallel. Although many rectangles have two long sides and two short sides, some do not. Therefore, having two long sides and two short sides is not a critical attribute of a rectangle. Squares share all the critical attributes of a rectangle but have the additional critical attribute of four equal sides.

Teachers should provide examples and non-examples of shapes so children can learn to categorize them.[85] A non-example of a shape lacks one or more critical attributes that define the shape. For instance, a long, thin rectangle is a non-example of a square because all the sides are not equal; a diamond (rhombus) is a non-example of a triangle because it has four sides instead of three. These and other examples and non-examples allow children to make distinctions about the basic features of shapes, paving the way for learning about relationships among shapes.

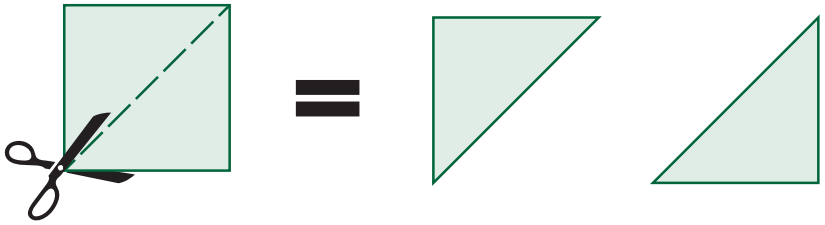
Once children are comfortable recognizing and comparing shapes, teachers should encourage children to explore how shapes can be combined and separated to form new shapes.[86] For example, two identical squares can be combined to form a rectangle, and a square can be cut along the diagonal to form two triangles or across the middle to form two rectangles, as shown in Figure 4.

Figure 4. Combining and separating shapes

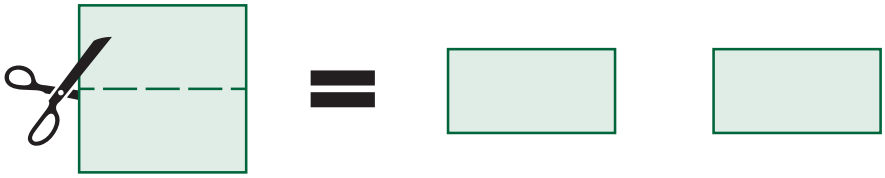
Two identical squares can be combined to form a rectangle.



A square can be cut along the diagonal to form two triangles.



A square can be cut across the middle to form two rectangles.



Exercises such as the *Shapes* game, outlined in Example 4, reinforce the properties of shapes and the spatial relations between them. When children manipulate shapes, they learn that changes in orientation do not affect the critical attributes of the shape.[87] They can also learn about spatial relationships between objects, such as "in," "on," "under," "beside," "above," or "below."

Example 4. The *Shapes* game

Objective

Identify and discuss attributes of various shapes and how to manipulate shapes to fit inside a larger field.

Materials needed:

- A large piece of poster board with a large shape drawn on it
- Various (precut) foam or plastic geometric shapes

Directions: Children draw from a basket or bag containing a variety of small shapes to put on the large shape drawn on a piece of poster board. The children take turns choosing a small shape from the basket and then identifying it, describing it, and placing it on top of the large shape. The group works together to fit as many small shapes as possible within the borders of the large shape without overlapping any of the shapes. When children have finished filling the large shape, they can count how many of each small shape they used and how many shapes were used in total. For subsequent games, the children can try to choose and place shapes strategically so the group can fit more small shapes inside the large shape. Teachers can tailor the *Shapes* game for use with the whole class, a small group, or individual children.

Early math content areas covered

- Geometry (shapes and attributes of shapes)

Monitoring children's progress and tailoring the activity appropriately

- Observe and note each child's ability to identify a shape and describe its attributes (number of sides, angles, and so on).
- Note children's ability to manipulate and place a shape strategically so the maximum number of shapes can be used.
- For inexperienced children, use only basic shapes (square, circle, triangle, and rectangle). As children become more proficient with the activity, increase the complexity of the shapes.

Integrating the activity into other parts of the day

- Blocks offer an opportunity for children to strategically manipulate and combine shapes to create other shapes and build more complex structures.

Using the activity to increase math talk in the classroom

- Talk about and describe shapes in the environment inside and outside the classroom.

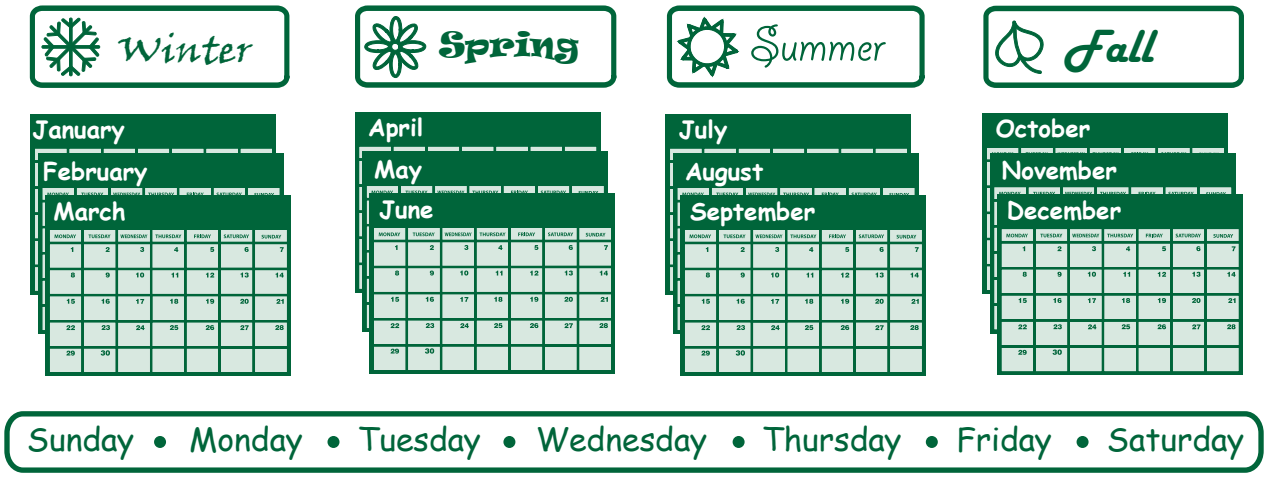
2. Encourage children to look for and identify patterns, then teach them to extend, correct, and create patterns.

Pattern instruction should begin by encouraging children to experiment with basic repeating patterns. For example, teachers can select a child to establish the pattern in which the rest of the class will line up for an activity (e.g., boy, girl, boy, girl, boy, girl). As children become familiar with simple patterns, they can experiment with more complex ones (e.g., boy, boy, girl, girl, boy, boy, girl, girl, boy, boy, girl, girl, as pictured in Figure 5).

Figure 5. Moving from simple to complex patterns

Teachers can encourage children to notice the patterns in the world around them, such as stripes on clothing, shapes and designs in rugs, planks in a wooden floor, or bricks on the sides of buildings.[88] Teachers can also describe the repetitive nature of the days of the week (Sundays are always followed by Mondays) and the number of months in a season, as displayed in Figure 6.

Figure 6. The repetitive nature of the calendar



Once children have become familiar with the nature of patterns, they should learn to predict what will happen next in a pattern, based on what has happened so far.[89] Children can use manipulatives, such as colored beads, to experiment with how patterns work. For example, teachers can create a string of alternating red and blue beads, and then instruct children to select the next bead in the string based on the current pattern. Teachers can also create errors in the previous pattern, such as two blue beads following a red bead,

and ask children to correct the errors. As children's understanding grows, teachers can provide opportunities for children to create patterns based on a set of instructions. For example, teachers could present the beads and strings to children and ask them to make a pattern in which two red beads follow every blue bead. Teachers can add complexity to the activities by introducing additional colors or other categories of beads based on size (big or small) or shape (round or square). Teachers can also encourage children to experiment and create patterns on their own, as outlined in Example 5.

Example 5. Creating and extending patterns

Objective

Recognize and create patterns of increasing complexity.

Materials needed:

- Short strings with a knot or fastener tied at one end
- Colored beads

Directions: Distribute short strings and handfuls of colored beads to the children. Create an example of a pattern, such as a red bead followed by a blue bead followed by another red bead. First, ask the children to recreate the existing pattern. Next, ask the children to predict which color will come next in the pattern. As the children's understanding grows, create patterns with deliberate errors (for example, following the blue bead with a second blue bead in the exercise above) and then ask the children to identify incorrect sequences. Finally, instruct the children to create patterns on their own. Teachers can tailor this activity for use with the whole class, a small group, or individual children.

Early math content areas covered

- Patterns

Monitoring children's progress and tailoring the activity appropriately

- Vary the number of beads to determine whether children are ready to use larger sets.
- If a child has difficulty, repeat the pattern several times in the same string of beads (e.g., red, blue, red, blue, red, blue). If the child grasps the exercise quickly, use more complicated patterns (e.g., red, blue, red, blue, blue, red, blue, blue, blue).

Integrating the activity into other parts of the day

- Adapt the exercise to include patterns children find in the world around them. For example, encourage children to look for patterns in the tiles on the classroom floor (square tiles and rectangular tiles), the bricks on the outside of the school (big bricks and small bricks), the clothing they wear (stripes, plaids, and other designs), or music they hear (verses and choruses).

Using the activity to increase math talk in the classroom

- Ask children to create patterns using themselves when lining up, and emphasize that a pattern is a repeating sequence.
- Blocks can provide children with an opportunity to create patterns while building structures.

3. Promote children's understanding of measurement by teaching them to make direct comparisons and to use both informal or nonstandard (e.g., the child's hand or foot) and formal or standard (e.g., a ruler) units and tools.

Teachers should show children how to compare objects for the purpose of sorting, arranging, and classifying them.[90] Teachers can help children understand what it means to compare the characteristics of two objects and identify similarities and differences. For example, as children's understanding of comparisons develops, children can begin to compare the lengths of two pieces of string to determine which is shorter or longer. Teachers can expand on this concept by demonstrating how to arrange a collection of pieces of string from shortest to longest. When making comparisons, teachers should reinforce measurement vocabulary words that describe the characteristics of the objects and the differences between them. Table 5 provides examples of vocabulary words associated with different types of measurement.

Once children have become comfortable making direct comparisons between and among objects, teachers can provide children with opportunities to measure objects using nonstandard tools and informal units, such as children's own hands and feet, or classroom items such as pencils, blocks, or books. After children learn to assign numerical values to the objects they are measuring with nonstandard tools (such as measuring the width of a table by counting how many "hands across" it is), they should be introduced to the concept of standard units of measurement (e.g., inches, feet, ounces, or pounds) as well as measurement tools (e.g., rulers and scales). Practice with these concepts can help lay the foundation for learning formal measurement vocabulary, tools, and techniques in later grades.[91]

By first using nonstandard measurement and then progressing to standard ways of measuring, children will discover that nonstandard measurements can vary, but standard measurements do not. For example, children could measure something familiar, such as the distance from the door to the writing center or the distance from the classroom

Table 5. Examples of vocabulary words for types of measurement

Type of Measurement	Examples of Vocabulary Words
Length 	long, longer, longest; short, shorter, shortest
Size 	small, smaller, smallest; big, bigger, biggest
Temperature 	warm, warmer, warmest; cold, colder, coldest
Time 	early, earlier, earliest; late, later, latest
Weight 	heavy, heavier, heaviest; light, lighter, lightest

to the restroom, by counting the number of steps between the two locations. Teachers could emphasize that children's measurements may vary depending on the size of the steps they take. Once children have learned to assign numerical values and use measurement vocabulary and tools, they can measure the distance in standard feet and inches using rulers and yardsticks.

Other opportunities for practicing measurement concepts include monitoring growth in height and weight, changes in temperature ("Today is warmer than yesterday") through different seasons, and differences in time ("We eat breakfast in the morning, and we eat dinner at night"). Children will learn that thermometers, scales, and rulers produce more consistent measurements than nonstandard tools. Understanding the numerical values associated with measurement will then help children make comparisons between objects. Children can utilize their existing knowledge of number to determine that an object with a length of 10 inches is longer than an object with a length of 5 inches because ten is more than five.

4. Help children collect and organize information, and then teach them to represent that information graphically.

Teachers should provide children with opportunities to count and sort familiar items to introduce them to the concept of organizing and displaying information.[92] This information can take the form of tangible objects, such as toys or blocks, or abstract concepts, such as characteristics (e.g., which children are 4 years old and which children are 5 years old) or preferences (e.g., favorite snacks, colors, or animals). The goal of such exercises is to demonstrate both the characteristics that distinguish the items and the total number in each set relative to other sets. For example, teachers could introduce sorting exercises when children are cleaning up and putting away toys. For children interested in building, teachers could encourage recording the number of different types of blocks. For children interested in drawing, teachers could encourage sorting, counting, and recording the number of crayons versus markers versus colored pencils.

Once children are familiar with sorting and organizing the information they have collected, they should learn to represent their information visually.[93] Graphs allow children to summarize what they have learned, and graphing provides an opportunity for children to share and discuss their findings.[94] Teachers can begin by introducing simple tallies and picture graphs to children, then teaching children to interpret the meaning of these graphs. Teachers can eventually move on to more complex graphs to illustrate changes in children's height or weight or to describe different characteristics of children in the class (e.g., gender, favorite color, clothing, or hair color). Example 6 describes a game in which children sort and discuss information with the class.

Example 6. The *Favorites* game

Objective

Have children practice sorting and grouping.

Materials needed:

- Signs for each sorting category, located in different areas of the classroom. In this example, children are sorting based on their favorite food.

Directions: Create a sign for each food, and place the signs in different areas of the classroom. Then, ask each child to share his or her favorite food with the class. Have the children find and stand near the sign that designates their favorite food. Once every child has joined a group, ask the children which food is the most common and which is least common.

Early math content areas covered

- Organizing and presenting information
- Number and counting

Monitoring children's progress and tailoring the activity appropriately

- Note each child's ability to name his or her favorite food, select the appropriate group, and answer questions about the information gathered.

Integrating the activity into other parts of the day

- Transition children by favorite food (e.g., "All the children who like apples can line up").

Using the activity to increase math talk in the classroom

- When children have sorted themselves, ask comparison questions such as "Which group has the larger/smaller amount?"

Potential roadblocks and solutions

Roadblock 2.1. *It is challenging enough to cover everything I need to cover in a day without having to think about four more early math content areas.*

Suggested Approach. Teachers may be able to find opportunities to cover more than one early math content area (number and operations, geometry, patterns, measurement, and data analysis) in the context of a lesson. For example, children can bring a collection of objects from home or find a collection of objects during recess. Children can first count the items in the collection and then arrange them in a pattern. Teachers can encourage children to identify any shapes in the collection and to name the critical attributes of those shapes. Children can be prompted to arrange the items according to characteristics such as size, length, weight, and so on. Finally, teachers can instruct children to sort their collections, compare the groups, and represent the information in a simple graph to identify which groups have more, fewer, or the same number of items. Addressing multiple math content areas within one activity might make it easier for teachers to cover all of the material assigned to that day. Another approach is to develop math games that can be played during transitions and down time that both help with classroom management and reinforce math concepts, particularly ideas that children have found challenging that week. For example, "I spy" games can be played anywhere and can be used to practice identifying shapes or patterns.

Roadblock 2.2. *Some children are struggling with basic vocabulary skills or are being exposed to English for the first time.*

Suggested Approach. Teachers can link visual representations of the most important vocabulary and concepts for geometry, patterns, measurement, and data analysis with terms in the child's home language, as well as in English, particularly when multiple children in the classroom speak the same language.[95] Teachers can help English-speaking children learn to count in their classmates' native languages to learn about each other. Songs and fingerplays are helpful tools for learning new words and math concepts. Using math manipulatives and inviting children to arrange materials or draw to show their answers can also help bridge the language gap.

Recommendation 3

Use progress monitoring to ensure that math instruction builds on what each child knows.

Evidence from studies of several math curricula suggests that preschool, prekindergarten, and kindergarten children are most likely to gain math knowledge when they are frequently exposed to targeted, purposeful, and meaningful math instruction.[96] Progress monitoring can be a useful way to ensure that children are receiving this type of instruction.

When developmental progressions (as described in Recommendations 1 and 2) are combined with progress monitoring, teachers can adapt lessons to a child's growing math knowledge. Effective instruction targets a child's developmental level (i.e., the child's skill level based on a developmental progression) and helps the child achieve the next level in the progression.[97] Connecting the information that is currently being taught to what children already know facilitates learning. By continually monitoring a child's progress, teachers can gather the information they need to match lessons to an individual child's knowledge level. Children develop knowledge at different times and at different paces.[98] Deliberately incorporating these individual differences into lesson planning by monitoring progress and tailoring instruction can help ensure that all children are encouraged to learn math concepts and skills that are appropriately challenging and just beyond their current level of understanding.[99]

Summary of evidence: Minimal Evidence

The panel assigned a rating of *minimal evidence* to this recommendation based on their expert opinion and 11 randomized controlled trials[100] and 1 quasi-experimental study[101] that met WWC standards and examined interventions that included at least one component of Recommendation 3. The studies supporting this recommendation were conducted in preschool, prekindergarten, and kindergarten classrooms.

The 12 studies examined curricula that included regular, short assessments during lessons. These assessments may have been informal, computer-based, or supported by rubrics to be used by the teacher during small-group instruction. Two interventions that included regular, short assessments were examined in six studies. Four of the six studies examined an intervention that included supports for assessments. On average, children who participated in the intervention scored higher on math outcomes than did children in the comparison condition.[102] Two of the six studies examined a number sense curriculum that included regular informal assessments to support the tailoring of review sessions. Once again, children who participated in the intervention tended to score higher on math outcomes than children in the comparison condition.[103]

Additionally, some curricula included "upward" and "downward" extensions of activities to support teachers in tailoring their instruction. The study examining *Pre-K Mathematics*, which provides both assessment tools and extension activities, found that children who participated in *Pre-K Mathematics* scored higher on average on children's general numeracy as measured by the Child Math Assessment (CMA) than children participating in the school's regular math instruction which may not have provided assessment tools and extension activities.[104]

The panel concluded that the body of evidence assessed in relation to Recommendation 3 was promising. However, it was not sufficient to warrant a moderate evidence rating as the panel was unable to definitively attribute the effects in the studies to the strategies included in Recommendation 3 due to two characteristics of the studies. First, the interventions examined in the studies were multi-component interventions that included strategies related to Recommendation 3 and other recommendations in the guide.[105] As such, it was difficult to determine whether the use of progress monitoring alone, or in combination with other program components, was responsible for the effects seen in math achievement. It is also possible that progress monitoring had no effect, and other components (or practices) were responsible for effects observed. Second, in most studies, the difference in the amount and type of progress monitoring the intervention and comparison groups received was not always specified,[106] and thus was not considered a direct test of a key component of the recommendation. Based on its expertise and the effects of interventions that include progress monitoring, the panel believes the studies generally support this recommendation despite the limitations to the body of evidence.

The panel identified three suggestions for how to carry out this recommendation.

How to carry out the recommendation

1. Use introductory activities, observations, and assessments to determine each child's existing math knowledge, or the level of understanding or skill he or she has reached on a developmental progression.

When employing progress monitoring, teachers should first gather specific information about each child's skill level to determine where to focus instruction. The panel suggests three primary methods of determining children's level of math understanding: introductory activities, observation, and formal assessments.

- **Introductory activities** involve presenting a new concept to determine how much of the activity children are able to do independently. For example, teachers can begin a small-group lesson on shapes by giving each child a bag of small shapes, including a triangle, a square, a rectangle, and other assorted shapes. If possible,

these shapes should differ in size and color for each child. After presenting a lesson on the different shapes, the teacher could ask younger children to name and compare the shapes in their bags, inquiring whether there are fewer blue circles or green triangles in the bag, which rectangle is the longest, or which circle is the smallest. Teachers could challenge older children to remove a shape from the bag—a rectangle, for example—and to tell the group how they know it is a rectangle. This kind of introductory activity can provide an opportunity for the teacher to assess a child's ability to sort shapes with similar features and classify them using math vocabulary.

- **Observation** involves using a math activity that addresses a specific skill and watching how children try to complete or solve the task. Often, watching children trying to solve a problem provides information about what knowledge they have and what knowledge they lack (see the *Monitoring children's progress and tailoring the activity appropriately* section of each Example for more progress-monitoring suggestions). Additionally, teachers can discover what children understand by asking them questions that require the children to think out loud and describe their problem-solving processes. Teachers can use these techniques to determine whether children are ready to move on to a more advanced concept or need more practice.

- **Formal assessments** typically occur at designated times of the year and can be standardized tests or other assessments that may not be chosen or administered by the teacher. Such tests can serve as screening and planning tools if used before or during instruction. If teachers receive feedback on children's performance on these assessments, they can use the information to plan activities and lessons. In addition to looking at total scores, it can often be useful to examine how children answer particular questions. It may be clear from some test sections that children are struggling with particular concepts, such as number recognition or counting. This information can help teachers direct their instruction to particular goals.

2. Tailor instruction to each child's needs, and relate new ideas to his or her existing knowledge.

Teachers should continually monitor a child's learning by employing a combination of strategies from the first step in this recommendation and should then use that information to design instructional activities.[107] Once teachers have information about a child's skill level, they can use a developmental progression to determine what the child should learn next and then can choose activities that are at or slightly above the children's level of understanding. For example, once a child can use small-number recognition to compare small collections, he or she can use meaningful object counting to determine the larger of two collections (for more details on a developmental progression for number and operations, see Table 3). Activities that are only slightly above the child's level of understanding can help ensure that the child does not feel frustrated by an activity that is too difficult. For example, knowing how many objects a child can successfully count in a set allows the teacher to gradually increase the number of objects so that the child can practice counting larger sets.

When tailoring instruction to individual students, the goal is not only to build on a child's existing math knowledge, but also to connect instruction to his or her interests in a variety of content areas. Relating new skills to children's existing understanding and experiences can help build knowledge. For example, if children have a particular interest

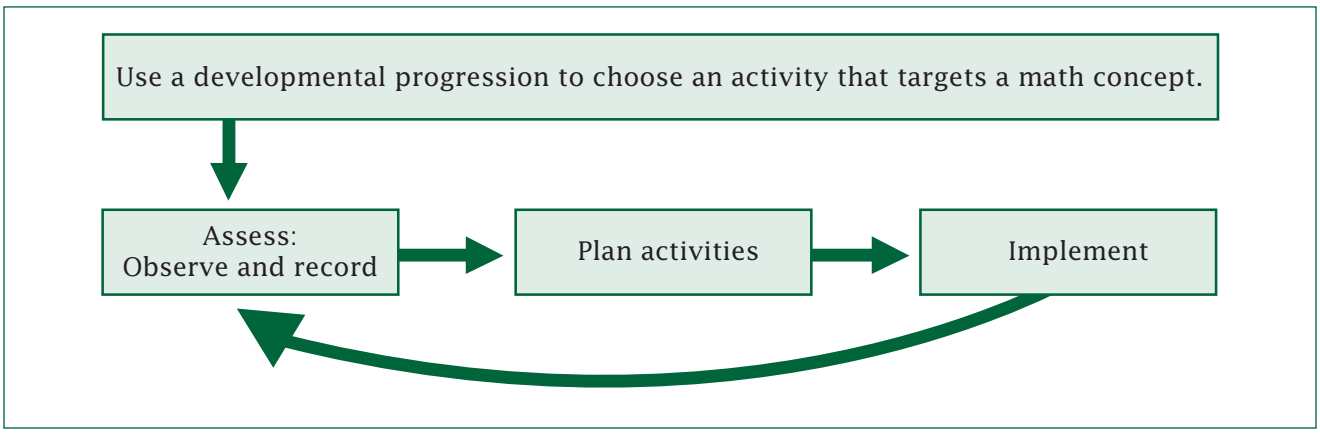
in music, teachers can design math activities that involve musical instruments. Children can determine how many instruments they need for everyone to play together or how many sticks are needed to play all the drums; they can count, sort, and compare different sets of instruments (how many drums, how many wind instruments, etc.); they can count along with musical beats, claps, or marching; and they can create musical patterns (e.g., one drum beat, two claps, one drum beat, two claps). By engaging children in activities that are interesting and applicable to their daily lives, children can connect skills across different activities and content areas.

Small-group activities can be a useful way of adapting instruction when children in a class are at different developmental levels and abilities. For example, using small-group time to play board games is one way that children of different abilities can make connections among their math skills. As the children learn more, teachers can adjust the game based on the children's level of understanding. For example, teachers can tailor a board game to different developmental levels by customizing the spinner. The teacher can first use a color-coded spinner that matches colored spaces on the board, so that children can use a spinner without numbers. The teacher can then introduce a spinner that has both dots (representing the number of spaces to be moved) and numerals. These types of materials can be changed throughout the year: early in the year, children can rely on color; later, they can count the dots on the spinner; and finally, they can use numerals to play the game. For more examples of using games to teach math concepts and skills, see Recommendation 5.

3. Assess, record, and monitor each child's progress so that instructional goals and methods can be adjusted as needed.

It is important to continually monitor progress so that children can be consistently engaged in activities that are neither too far below their level (and therefore not interesting) nor too far above it (and therefore frustrating). Progress monitoring also allows teachers to plan what children should learn next. Example 7 contains a model of the flow of progress monitoring. In this model, a teacher focuses small-group instruction on counting small collections. The teacher observes and records the children's progress using the checklist in Example 8. Looking at the largest set counted successfully and the type of errors made, the teacher can plan different activities for the two children, Sarah and Bill. Sarah should continue counting small collections, while Bill is able to move on to comparing magnitudes of collections. The teacher should also plan to reassess Sarah and Bill, repeating the ongoing process of progress monitoring.

Example 7. The flow of progress monitoring



While engaging in progress monitoring, teachers may want to keep track of their observations, as shown in Example 8. While children are involved in a math activity, the teacher can observe and quickly note what each child can and cannot do. By keeping a record of children's skill progression, teachers can more easily determine where a child may need extra help or what activities the child can do particularly well. For example, a teacher can observe a child counting objects to assess whether the child can successfully count with one-to-one correspondence. If the teacher notices a child making a coordination or sequencing error, the teacher can note the type of error to help determine which activities the child should work on next to practice this skill. (See Table 4 for common counting errors.)

Example 8. Progress-monitoring checklist

Activity: How many stars are there? (Child is asked to point and count "how many stars.") 	**Child**	**Date**	**Activity**	**Largest Set Counted Successfully**	**Types of Errors Made**
	Sarah	September	counting stars	5	skips "six" when counting
	Bill	September	counting stars	10	sometimes double-counts a star

Potential roadblocks and solutions

Roadblock 3.1. *How can I maintain order in the classroom when breaking the class into small groups?*

Suggested Approach. When children are in small groups, classroom behavior can sometimes become chaotic and noisy. There are three things to think about when forming small groups. First, group children strategically to avoid social conflicts. If children of mixed abilities are working together in groups, ensure there is the right mix of ability levels. Second, develop activities that build on children's interests. Using small groups enables teachers to present more challenging activities to some children so that they do not become bored. Finally, plan adult assistance to facilitate independent and adult-supported learning for all groups. One strategy for managing groups is to use round-robin learning centers.[108] While one group is meeting with the teacher, other groups are productively engaged in different learning centers. There should be one center for each group not meeting with the teacher. The teacher then is free to focus on one small group at a time.

Roadblock 3.2. *I am already required to give standardized assessments. Can I use my existing assessments to tailor instruction?*

Suggested Approach. Teachers can review the assessments to find questions that apply to the particular skill they would like to target. Then, they can use those questions to gauge where children are and at which level to target activities. If teachers receive feedback on how children in their classroom are performing on a standardized assessment, they can fit this feedback in with the developmental progressions to determine which areas need more focus and when children can move on to higher-level skills. If there are assistant teachers, aides, or other adults available in the classroom, teachers can ask to them to share in observing children and keeping brief checklists of children's ability levels.

Roadblock 3.3. *What if I do not have required assessments, or the assessments do not fit well with the skills that are targeted in the developmental progression?*

Suggested Approach. Teachers can use a developmental progression to develop an activity that will provide information about the child's skill level. For example, teachers can develop a checklist of numerals from 1 to 20 and use magnetic numerals or a numeral bingo game to assess the child's ability to recognize numerals. Teachers can generate checklists for counting collections, naming shapes, identifying patterns, sorting, and many other math skills.

Representations are objects, actions, words, pictures, or symbols that stand for ideas.

Teach children to view and describe their world mathematically.

Teachers can encourage children to look for opportunities to describe math ideas in the world around them, gradually moving from informal representations and language to formal representations and math vocabulary as children's understanding grows.[109] By exploring their environment and interacting with manipulatives, children can begin to apply their math knowledge.[110] At first, children should use informal tools such as their fingers, tally marks, or other concrete objects to represent math ideas. For example, children can be encouraged to use blocks to model and solve simple addition problems (e.g., "If I have two blocks, and I add three more, how many blocks do I have?"). Once children are comfortable using math informally, teachers can help them link their informal knowledge to more abstract math concepts, formal math vocabulary, and formal representations such as math symbols.[111]

If children hear math vocabulary in context and then practice using it, they may be better able to understand the underlying math concepts.[112] The panel believes there is evidence of a positive relationship between math-related talk and children's math knowledge.[113] As one part of math-related talk, teachers can use open-ended questions to prompt children to think about how to describe their ideas mathematically and to increase the amount of math-related dialog in the classroom. If a child can describe his or her method for solving a problem to someone else and then hear other children describe their approach to a problem, all the children may learn to apply their math knowledge in new ways.[114] Teachers can reinforce this idea by encouraging children to look for opportunities to use their developing math skills throughout the school day.

Summary of evidence: Minimal Evidence

The panel assigned a rating of *minimal evidence* to this recommendation. The rating is based on their expertise and 14 randomized controlled trials[115] and 2 quasi-experimental studies[116] that met WWC standards and examined the effects of interventions designed to help children view and describe their world mathematically. The studies supporting this recommendation were conducted in preschool, prekindergarten, and kindergarten classrooms.

Some interventions provided specific math vocabulary words[117] and suggestions for stories,[118] songs, or questions[119] that supported children in learning to view and describe their world mathematically. Studies examining these interventions found positive effects in the general numeracy, basic number concepts,

and geometry domains.[120] In two studies, math conversation, whether with a peer or an adult, resulted in higher math achievement.[121]

The panel concluded that the body of evidence assessed in relation to Recommendation 4 was promising. However, it was not sufficient to warrant a moderate evidence rating as the panel could not attribute the effects solely to Recommendation 4 for two reasons. First, the examined interventions were multi-component interventions incorporating elements of other recommendations in the guide.[122] Second, in some studies there was a lack of clarity regarding the instruction the intervention and comparison groups received.[123] Based on its expertise and the effects of interventions that include efforts to teach children to view and describe their world mathematically, the panel believes the studies generally support this recommendation despite the limitations to the body of evidence.

The panel identified four suggestions for how to carry out this recommendation.

How to carry out this recommendation

1. Encourage children to use informal methods to represent math concepts, processes, and solutions.

Math instruction for young children should begin with informal representations of math ideas.[124] Initially, teachers should link math ideas to familiar experiences, terms, or analogies, resisting the urge to use more formal methods until children have a conceptual foundation for understanding them.[125] For example, teachers should use terms that represent children's informal understanding of addition, such as "more" and "all together," as opposed to the more formal, symbolic representation. An example of informal understanding might be "Bill had three carrots, and his mother gave him one *more*. How many carrots does Bill have *all together* now?" This phrasing is in contrast to formal representations, such as "Three plus one equals what?" or "3 + 1 = ?" Table 6 provides examples of how to teach informal representations of math concepts.

Table 6. Using informal representations

Concept	Informal Representation	Teaching the Concept
whole number	"three"	Collections of blocks, dots, tally marks, fingers, or other countable objects can represent numerals. For example, when playing a game, use blocks to represent children's scores so everyone can track each player's score.
equal	"same number as" or "same as"	Provide opportunities for children to begin to recognize that collections that have the same number when counted are equal. For example, a collection of four plates is the same number as a collection of four cups.
unequal	"more than" or "fewer than"	Point out that a collection is more (or fewer) than another if it requires a longer (or shorter) count. For example, seven is more than six because it requires counting beyond six.
addition	"and" or "more"	Start with a collection and add more items to make it larger. For example, start with three crayons and add one more. Then ask, "How many?"
subtraction	"take away" or "fewer"	Start with a collection and take away some items to make it smaller. For example, start with three crayons and take away one. Then ask, "How many?"

2. Help children link formal math vocabulary, symbols, and procedures to their informal knowledge or experiences.

Once children are comfortable using informal methods and representations to describe math ideas, teachers can introduce math vocabulary and formal representations. Teachers should explicitly teach children math words so they have the vocabulary needed to connect their informal knowledge to formal terms.[126] Teachers can start with informal vocabulary and then connect these familiar terms to formal terms. For example, teachers might begin with the informal phrase "take away" and then later explain that "subtract" has the same meaning.

Teachers can then use this math vocabulary when speaking to children throughout the day. Vocabulary that is used during math instruction does not need to be restricted only to math activities. For example, words such as "more" and "fewer" can be emphasized throughout many different topics and activities. Math conversations can happen spontaneously as teachers comment about natural occurrences that involve number or other math concepts. For example, teachers can make a comment about which child is standing "first" in line or which child has "more" or "fewer" objects than another child. As another example, while the child is drawing a picture of his or her family, a discussion could focus on the "number" of family members and who is "older" or "younger."

Just as children learn to link math vocabulary to their informal knowledge, they should also learn to connect formal representations to their informal math knowledge. Linking formal representations to informal concepts and representations enables children to understand and more readily learn formal terms, symbols (e.g., + or –), definitions, and procedures.[127] For example, teachers can connect numerals to both quantities (e.g., a collection of five buttons) and verbal representations (e.g., the word "five").[128] Table 7 provides examples of lessons for linking familiar concepts to formal symbols.

Table 7. Linking familiar concepts to formal symbols

Symbol	Concept	Lesson
numerals	counting	Have children count and record the number of children in attendance each day.
+ , –	operations	Have children solve problems involving adding or subtracting with leaves collected from the playground.
=	equal	Show the class four pennies. Next, show three pennies, verbally label them ("I have one, two, three pennies"), and put them in a can. Then, show one more penny, verbally label it ("I have one more penny"), and put it in the can. Ask the class, "Are three pennies and one more penny the same number as four pennies?"
< , >	unequal	Show the class five pennies, verbally label them, and put them in a can. Next, show four pennies, verbally label them, and put them in a different can. Ask the class, "Which can has more? Which can has fewer?"

3. Use open-ended questions to prompt children to apply their math knowledge.

Open-ended questions can help children to develop cognitive and language skills. They prompt children to think through their actions, describe their thoughts, and learn from one another. Questions that begin with "what," "why," or "how" can encourage children to use math vocabulary to explain what they have learned. Teachers should ask questions that require children to use math-related terms to describe something. For example, asking, "How can we find out (how many children are here today, how much snack we need, etc.)?" gives children the opportunity to communicate about a math strategy and then to practice that strategy. The questions can be tailored to current math objectives. See Table 8 for examples of questions teachers can ask that are related to the math content areas.

When asking open-ended questions, teachers can employ techniques to encourage math-related conversation. First, before calling on a child, teachers might allow enough time for more than just a few children to think of an answer. When in groups, one child can help another child come up with an answer. Rather than saying "yes" or "no" quickly, teachers can allow multiple possibilities to be discussed. For example, a teacher can show the entire class a picture of a mother and a daughter holding hands, waiting for the school bus. The teacher can ask "How are these two people different?" One child may answer, "The mother is bigger than the daughter." Another child may answer, "The mom is wearing stripes and the daughter is wearing dots." Although the teacher should ultimately focus on the correct answers that best fit a math context, he or she should acknowledge that there are multiple correct responses.

Table 8. Examples of open-ended questions

How are these the same/different?
What can you use (in the block area) to make a pattern?
What patterns do you see (on the seashells in the science center)?
How could we change this pattern to make a new one?
How can we find out who is taller or shorter?
What can we use to find out…?
What can we do to find out who has more/fewer?
How else can you show it?
How does it show what we know?

4. Encourage children to recognize and talk about math in everyday situations.

Teachers can encourage math thought and conversation by asking children for their help with problems that arise throughout the day.[129] For example, a teacher might say, "I have to figure out how many cups we are going to need for the birthday party. Can you help me? How should we do that?"

Once children solve the problem, teachers can have them describe their method by asking a sequence of questions that prompts the children to share the solution and the strategies used to reach the solution. For example, if the problem involves how many orange slices are needed for snack time, the teacher could ask the children for an answer. Then, the teacher could say, "How did you figure that out? What did you do first? Then what did you do?" During small-group time, the teacher and children could have a more formal discussion about the steps used to solve the problem.

After a child shares his or her solution, the teacher might repeat the problem-solving steps back to the child in sequence to continue the math talk. For example, the teacher could say, "Oh, I see, first you counted how many children were here. Then you thought about how many orange slices each child might eat." To continue the conversation even further, the teacher could ask the group, "Is there another way you can do that?" or "How else could we do this?"

When children are given explicit math problems to solve, it can be helpful for them to talk through their problem-solving process.[130] For example, in an extension of the *Basic Hiding* game in Recommendation 1, when a child successfully tells how many objects are hidden, the teacher can ask the child to describe how he or she knew how many there were. It is important to keep children's developmental levels in mind. At first, many children may not be able to describe their problem-solving process. Teachers can aid children by talking through their own problem-solving strategies out loud, demonstrating for the children how to use math vocabulary when describing their thought processes. As teachers help them with the math conversation and emphasize the math vocabulary (e.g., "There were five blocks, and then I added three more blocks"), teachers can help children begin to develop the skills they need to communicate about the problem solving that they or their peers are doing.

Potential roadblocks and solutions

Roadblock 4.1. *I'm not sure what types of open-ended questions are most effective for getting young children to think mathematically.*

Suggested Approach. Teachers can start a lesson with "What do you think?" or "How can we find an answer?" When children give an answer, teachers might ask, "How did you figure that out?" or "Show me how you did that." If children share a strategy, teachers might also ask, "Is there another way to solve that problem?" or "What would happen if I changed…?" Asking children to compare and contrast also helps them clarify their ideas ("How are these [shapes, numbers, patterns, measuring] tools alike or different?"). These questions are appropriate for any math content area.

Dedicate time each day to teaching math, and integrate math instruction throughout the school day.

Dedicated time that is devoted to planned, daily math lessons can allow children to develop important skills in number and operations, geometry, patterns, measurement, and data analysis. By connecting math to a variety of everyday situations and routines, teachers can make math meaningful and provide opportunities for children to practice what they have learned in a purposeful manner.[131] If teachers coordinate their current math objectives with activities in the classroom and lessons in other subject areas, children can master skills and extend the concepts to higher levels or broader contexts.[132]

A classroom environment that contains math-related objects can help children recognize and apply math knowledge. For example, games can provide an enjoyable and meaningful way to learn a range of math ideas and practice a wide variety of basic skills.[133] Games can build on children's math knowledge, provide a reason for learning skills and concepts, supply repeated practice that is not boring, give children and teachers an opportunity to discuss strategies and ideas, and generate excitement.[134]

Summary of evidence: Minimal Evidence

The panel assigned a rating of *minimal evidence* to this recommendation. The rating is based on their expertise and 18 randomized controlled trials[135] and 2 quasi-experimental studies[136] that met WWC standards and examined the effects of interventions that included dedicated time for math instruction, integration of math into other aspects of the school day, and use of games to practice math skills. Children in the studies attended preschool, prekindergarten, and kindergarten.

One of the studies examined *Math Is Everywhere*, a collection of 85 suggested activities (e.g., books, music, games, discussions, and group projects) that reinforce math concepts.[137] These activities can be implemented during various times of the day, such as circle time, transitions, or mealtimes. Children in classrooms using *Math Is Everywhere* scored higher in the general numeracy domain than children in classrooms where the teachers continued their regular classroom instruction. These higher scores could be due to teachers providing daily math lessons and incorporating math into various times of the day; however, the scores could also be due to aspects of other recommendations present in the intervention.

Another group of studies found that children who played number-based board games performed better in the domain of basic number concepts than did children who

played color-based board games or no board games.[138] However, the effects of number-based board games on measures of number recognition and operations were mixed.[139] The interventions in which playing a board game was part of a larger curriculum included not only elements of this recommendation but also other recommendations in the guide.[140]

The panel concluded that the body of evidence assessed in relation to Recommendation 5 was promising. However, the panel identified two limitations to the body of evidence. First, the examined interventions were multi-component interventions incorporating elements of other recommendations in the guide.[141] Second, in some studies there was a lack of clarity regarding the instruction the intervention and comparison groups received.[142] Despite these limitations, the panel recommends dedicating time to teach math, integrating math into other aspects of the day, and using games to practice math skills based on its expertise and the pattern of positive effects.

The panel identified five suggestions for how to carry out this recommendation.

How to carry out this recommendation

1. Plan daily instruction targeting specific math concepts and skills.

In order for preschool, prekindergarten, and kindergarten children to develop math skills, teachers should set aside time each day for purposeful math instruction.[143] Dedicated time for math instruction can help to provide children with skills in the foundational areas of math described in Recommendations 1 and 2. During math lessons, teachers can help children learn specific skills they can build upon throughout the rest of the day (as described in the remainder of this recommendation). Teachers can use large and small groups during dedicated math time to tailor instruction for children at different developmental levels.

Large-group (or whole-class) time can be a good place to introduce a concept for the first time or illustrate a concept through an example that is relevant to children's everyday lives. For example, teachers can read children a book that relates to the skills that will be taught, or they can play a whole-group game with the class. It is important to remember, however, that introducing a concept in a large group is most helpful when children have similar skill levels; it is also useful to reinforce the concept in smaller groups, particularly for children whose math understanding may not be as advanced as other children and who may miss key instructional points during whole-group activities.

After a particular concept is introduced in a large group, teachers should provide time for at least one small-group activity to help children practice and reinforce their skills. It may be particularly useful to broadly introduce a math concept during a large-group time, then tailor instruction to small groups of children who are at similar developmental levels so they can work on particular aspects of that skill, as described in Example 9.

2. Embed math in classroom routines and activities.

A daily or weekly schedule provides many opportunities to reinforce math concepts outside of the dedicated math instruction period.[144] Routines such as taking attendance can serve a math purpose in addition to a practical one. For example, teachers can engage children in using tally marks, beads, abacuses, or other markers to count how many girls, boys, and total children are in the classroom. After the count is decided, the teacher can extend math thinking

Example 9. Linking large groups to small groups

Objective

Understand the differences and similarities between triangles, rectangles, and squares.

Materials needed:

- Book: *Bear in a Square*, by Stella Blackstone
- A variety of other objects (based on availability, but could include the following)
  - Large pieces of paper cut into varied shapes for painting
  - Lunch trays and a small amount of sand
  - Geoboards with rubber bands

Directions, large group: Read the book in a large group, highlighting the names of all the shapes but focusing specifically on the difference between the number and length of sides and types of angles in triangles, rectangles, and squares.

Directions, small group: Once children are divided into small groups, highlight the number and length of sides and types of angles in each of the shapes the children create in the activities below. Children should be encouraged to use informal terms to describe the shapes, such as "long" and "short" sides and "big" and "little" angles for triangles. These activities will vary based on the types of materials available, but they could include the following:

- Provide paint, chalk, or other art materials so that children can add a stripe around the edge of a large paper cutout of a triangle or rectangle. Then, have the children continue to add more of the same shapes inside the original shape to create a design with concentric shapes.
- Lead children to use their fingers to draw shapes in sand on a tray or in a sandbox. They might draw shapes within shapes or combine shapes to make other figures.
- Encourage children to experiment with placing rubber bands on a geoboard to make triangles, rectangles, and squares of different sizes and orientations.

Early math content areas covered

- Geometry (shapes and attributes of shapes)

Monitoring children's progress and tailoring the activity appropriately

- For children who are more advanced, more complex shapes can be used. More advanced children may notice the number of sides on other shapes, such as a pentagon, or may ask about the number of sides in a circle.

Integrating the activity into other parts of the day

- Take a group walk outside to collect sticks of different sizes, and then use them to make and identify shapes.

Using the activity to increase math talk in the classroom

- Encourage the children to look around their environment, such as on tables in the classroom or on their clothing, to identify examples of triangles, rectangles, and squares. When children locate a shape, ask them to explain it to the group: "How can you tell that shape is a ___?" Prompt the children to identify the number and length of sides and type of angles.

by saying, for example, "We have 8 girls and we have 10 boys. We have 18 children all together: 8 plus 10 equals 18." The class could then display the results of attendance for several days using a chart that has columns or rows titled with the days of the week or a pie chart with the number of slices in the pie matching the total number of children in the class on a particular day. Teachers can also engage children in other everyday activities that may have a math component. For example, teachers can have children answer a yes/no "question of the day" every day. Children can then record how many of their classmates said "yes" and how many said "no" in a graph and compare the two numbers. Example 10 describes an opportunity to reinforce math concepts during snack time, another routine activity.

Example 10. Snack time

Objective

Practice counting, cardinality, addition, and subtraction.

Materials needed:

- Snacks
- Plates or paper towels

Directions: Once children receive an equal number of snacks, have them count how many they have. As they eat their snacks, they can compare how many they have relative to other children. Teachers can tailor snack time activities for use with the entire class or small groups.

Early math content areas covered

- Counting using one-to-one correspondence
- Cardinality
- Adding and subtracting (one more/fewer)

Monitoring children's progress and tailoring the activity appropriately

- Observe and note how each child counts the snacks. For example, does the child line up the pieces of the snack, or can the child count the pieces while they are scattered?
- Adapt this activity for children of varying levels by reducing the number of snack pieces to count or by asking each child different questions, such as "How many will you have after you eat one?" or "How many will you have after your friend gives you one?"

Integrating the activity into other parts of the day

- Ask children, "How many?" and "How can we find out how many?" whenever the opportunity arises. For example, ask, "How many books did you read?" or "How many children built this beautiful tower?"

Using the activity to increase math talk in the classroom

- Ask children to count out loud and compare amounts throughout the day to increase math talk in the classroom.

3. Highlight math within topics of study across the curriculum.

Teachers can integrate math concepts into non-math lessons by highlighting the aspects of math that are already present in the curriculum.[145] Teachers can point out opportunities to count objects, examine shapes, analyze data, or measure objects (depending on the current math objectives and where children are in the developmental progressions for these content areas).

During literacy time, for example, when reading a story, the teacher can ask questions that encourage children to solve a math problem based on the story. If the class is reading a story about the three little pigs, the teacher can ask the children to count the pigs, or the teacher could ask how many cupcakes they would need for a party with the three pigs. Teachers should select books that reinforce current math objectives. Teachers can also consider using more than one book to illustrate a given math concept, so children understand that a concept or skill can be applied in multiple contexts. Table 9 provides examples of ways to integrate different math content areas into literacy, science, art, health and safety, and social studies lessons.

Table 9. Integrating math across the curriculum

	Math Content Area				
	Number and Operations	**Geometry**	**Patterns**	**Measurement**	**Data Analysis**
Literacy	*We All Went on Safari*, Krebs *Mouse Count*, Walsh *7 Little Rabbits*, Becker and Cooney	*Bear in a Square*, Blackstone *Mouse Shapes*, Walsh *Shapes*, Silverstein	*A Pair of Socks*, Murphy *Pattern Bugs*, Harris *Pattern Fish*, Harris	*How Big Is a Foot?*, Myller *Spence Is Small*, Chevalier *Tall*, Alborough *The Grouchy Ladybug*, Carle	*It's Probably Penny*, Leedy *The Great Graph Contest*, Leedy *Tiger Math*, Nagda and Bickel
Science	Count collections of natural objects. Count how many days it takes for a plant sprout.	Describe objects from nature (e.g., rocks, leaves, and insects) in geometric terms. Use precut shapes to make animals.	Find and identify patterns in nature (e.g., on butterflies and snakes). Design a model of an insect using a pattern design.	Measure the growth of a plant in the classroom each day and predict how much time it will take before flowers will be visible on the plant.	Graph the amount the classroom plant grows each day. Graph animals with two legs, four legs, and more than four legs.
Art	Count how many objects appear in a piece of artwork.	Identify shapes in artwork. Decorate drawings of shapes.	Use patterns to make pictures or frames for pictures. Find and identify patterns in artwork.	Use measurement to make frames for art out of poster board or card stock.	Make a graph of the children's favorite colors. Tally children's opinions about artwork. For example, ask, "Which painting do you like better?"

Table 9. Integrating math across the curriculum *(continued)*

	Math Content Area				
	Number and Operations	**Geometry**	**Patterns**	**Measurement**	**Data Analysis**
Health and Safety	Count the length of time it takes to wash your hands. List rules for washing hands or playing safely outside.	Use traffic signs to recognize shapes. Walk lines that are different shapes to practice balance control.	Jump rope or play hopscotch with an alternating pattern.	Measure your body's growth over time.	Graph your height or foot size.
Social Studies	In a unit about families, order people by size or from youngest to oldest. During a unit on recycling, children can count how many of a certain object they have collected to recycle.	Identify squares, straight lines, curved lines, etc., on maps.	Study patterns in clothes from different parts of the world. Look for patterns in flags from other countries.	Make a map of the neighborhood using measuring, geometry, spatial thinking, and positioning words.	Graph the size of the children's families. Make a graph that shows how children come to school (by bus, by car, etc.).

4. Create a math-rich environment where children can recognize and meaningfully apply math.

Teachers can provide opportunities for children to see and use math concepts regularly by creating a math-rich classroom environment. This enrichment can be done by making math-related objects and tools readily available, labeling and organizing math-related objects and tools so they are easy to find and use, and organizing activities and routines with numeric systems.[146]

Teachers should provide a variety of tools throughout the classroom to allow children to explore each of the five math content areas. Table 10 lists examples of tools for different math content areas.

Table 10. Examples of tools that can be useful in each math content area

	Number and Operations	**Geometry**	**Patterns**	**Measurement**	**Data Analysis**
Objects and Tools	blocks abacuses number lists number puzzles	geoshapes precut foam shapes traffic signs for classroom areas	beads different-colored cubes art materials, such as stamps and markers	rulers tape measures clocks scales measuring spoons and cups	clipboard and paper for tallying the "question of the day" hula hoops or small hoops that bend for Venn diagrams sorting bins

Teachers can explicitly teach children how to use tools by modeling their use during small- or large-group time.[147] For example, the teacher can use shapes or blocks to demonstrate how a rectangle and a triangle can be combined to make a house. As another example, the teacher can bring different types of measuring tools to circle time to demonstrate how to use tools to measure objects of varying sizes (e.g., placing the ruler next to the object to be measured, with the end of the ruler at one end of the object, then reading the number closest to the opposite edge of the object).

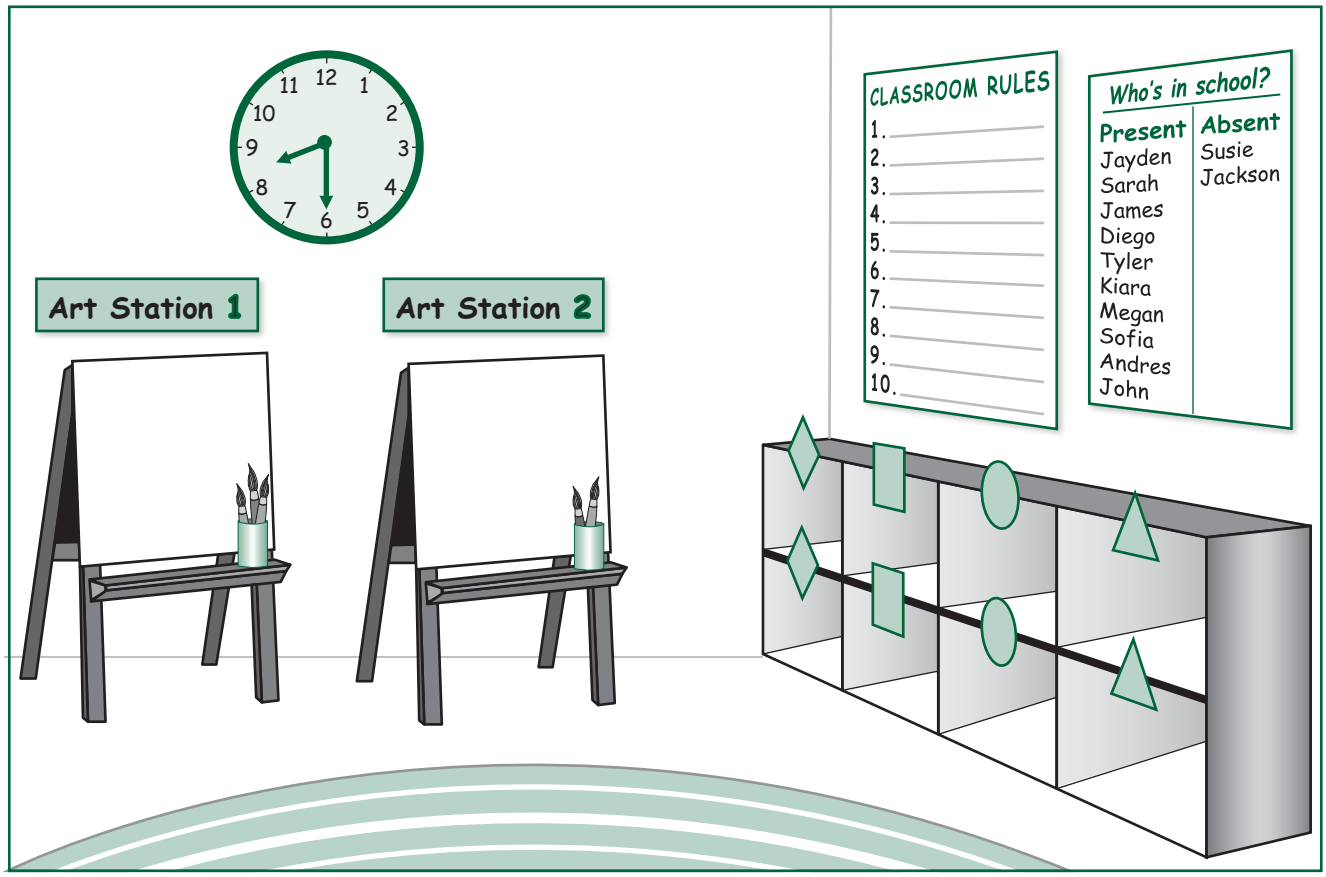
Teachers can place tools, such as number lists, rulers, and scales, at eye level for children. Also, the classroom can be organized and labeled in a manner that supports learning. For example, lunch tables can be labeled with shapes, and children can sit at the "triangle table" or the "circle table" for lunch.

After children learn those shapes, the labels can be changed to new shapes. These activities, along with activities described in Recommendation 4, help children learn and apply math vocabulary in meaningful ways.

Organizing activities and routines with numeric systems can give children opportunities to reinforce and practice math concepts while becoming more independent. To do this, teachers can display charts with sequenced directions and picture icons, number the classroom rules on a poster, or use a numerical system to indicate how many children can work in a center at the same time, as displayed in Figure 7.

Teachers can also involve children in labeling and organizing the environment as much as possible. For example, teachers can discuss how many people can safely work in a particular center, then have the children help make the label and the number for it.

Figure 7. An example of a math-rich environment in the classroom



5. Use games to teach math concepts and skills and to give children practice in applying them.

Games can provide an engaging opportunity to practice and extend skills. If children have fun playing the games, they are more likely to be motivated to practice math.[148] For maximum benefit, teachers should select specific games to match current math objectives. Example 11 provides an example of a game (*Animal Spots*) that reinforces one-to-one correspondence and cardinality. Games that target different math content areas are often included in math curricula. Games can also be purchased separately or be made by the teacher. Some math concepts may also be highlighted in games that come up during natural play, such as hopscotch or jump rope.

Example 11. The *Animal Spots* game[149]

Objective

Practice one-to-one correspondence and cardinality

Materials needed:

- Pictures of animals or materials children can use to draw their own animals
- Small circles of paper to use as spots
- Glue
- A die or spinner to determine the number of spots to place on each animal

Directions: Have each child draw the outline of an animal on a piece of paper, or provide handouts with large outlines of animals. Each child should take a turn throwing the die to determine how many spots to place on his or her animal. The children should count out the number of dots on the face of the die, and then they should choose the same number of "spots" from a bowl of paper circles in the center of the table. After children have selected the correct number of spots, they can glue them onto their animals. Teachers can tailor the *Animal Spots* game for use with the entire class, a small group, or individual children.

Early math content areas covered

- Counting using one-to-one correspondence
- Cardinality

Monitoring children's progress and tailoring the activity appropriately

- Observe the play, noting each child's ability to count the number of dots on the die and count out the same number of spots from a larger pile.
- Use one die or a spinner at the beginning; then, use two dice to increase difficulty.

Integrating the activity into other parts of the day

- Have children count out objects from a larger set. For example, a child can choose ten blocks for building or five shapes from a larger collection to use for a collage.

Teachers can get involved with the game-playing to ensure educational play. For example, if children are playing a game to learn one-to-one correspondence and cardinality, the teacher can emphasize moving one space at a time and then reinforce the total number of spaces that the game piece should be moved. The teacher can also use the game to extend children's skills. For example, if children are ready, the teacher can use a pair of dice instead of a single die or a spinner, so children have to count and add the dots on each die.

Potential roadblocks and solutions

Roadblock 5.1. *The school is on a limited budget and cannot afford to purchase many classroom materials or games.*

Suggested Approach. Math can be embedded in the classroom without spending a lot of money if teachers take advantage of opportunities that occur naturally throughout the day. For example, teachers can highlight math concepts that come up in an already-planned literacy or science lesson by asking children a question that requires them to use math concepts. In addition, teachers may be able to create games on their own with readily available natural materials such as leaves, sticks, and rocks.

When purchasing materials, strategic planning can help save resources. Teachers can choose games that teach the math content areas children are most interested in. They can also choose games that are accessible to a range of skill levels to avoid having to purchase more than one game. For example, if the teacher is playing a memory game with younger or less advanced children, the group can play with all the cards face-up, or they can play with fewer cards than the whole set. The teacher can play the same game with older or more advanced children by flipping the cards over and using the whole set.

Teachers can also turn to existing community resources. For example, they can take advantage of the local public library to find math-related books for their classroom. Many librarians can help teachers by selecting books related to certain topics requested by the teacher. Also, some communities may also have toy-lending libraries from which teachers may borrow games or other manipulatives.

Roadblock 5.2. *I am told that it is important to provide literacy-, science-, art-, and math-rich environments. It is difficult to keep all subjects in mind at all times.*

Suggested Approach. Teachers do not need to include all aspects of all subjects at one time. Instead, they can rotate the activities and materials in the classroom based on the instructional objectives at that particular time. They can also try to coordinate the use of materials and activities to meet multiple goals. For example, reading a story that contains math content areas can help meet a math objective and a literacy objective simultaneously. When lesson planning, teachers can select ahead of time the learning objectives they would like to focus on each day and then plan activities and modify the classroom environment to support those objectives.

Roadblock 5.3. *I do not have my own space because multiple classes use the same classrooms throughout the day.*

Suggested Approach. If the classroom environment cannot be modified, teachers should take advantage of ways to embed math concepts that do not involve modifying the classroom environment. Alternatively, teachers could use a mobile chart stand to hold multiple charts that could be displayed throughout the day.

Roadblock 5.4. *Parents may wonder why their children are playing games in school.*

Suggested Approach. Teachers should help parents understand the importance of play in motivating children to practice concepts they are learning in more formal math instruction. Teachers can help alleviate parental concerns by selecting games with certain objectives in mind, so when a parent asks why a certain game is being played at school, teachers can respond accordingly. For example, a teacher might say, "We are playing *Go Fish* because it helps the children recognize numbers, match numbers, and determine, at the end of the game, who has more matches and who has fewer matches." Teachers can also use board games to support children in learning numbers and counting. For an example of a game that a teacher could make, see Siegler and Ramani (2009).

Glossary

A

An **assessment** provides information on how much a child knows about a particular topic or the skills a child has in a particular area. Assessments may include an adult's observation of a child in classroom activities, an adult's rating of the child, or an adult's scoring of a child's accuracy on a particular task (e.g., test or worksheet). Assessments may be *formal*, such as standardized tests, standardized rating scales, teacher-developed tests, or worksheets. Teachers may also conduct *informal* assessments to check to see what a child knows or can do. Assessments can be *formative*, with the results used to determine the extent to which the child learned the intended skills from instruction as part of progress monitoring. Finally, assessments may be *summative*, with the result documenting a child's performance, for example, on an end-of-chapter test or state developed test. The particular type of assessment (formal or informal, formative or summative) should be chosen based on how the results will be utilized.

C

Cardinality is the total number of items in a collection. The **cardinality principle** is the understanding that when counting, the number word assigned to the last item of a collection represents the total quantity.

A **collection** is a group of discrete objects or things.

D

A **developmental progression** refers to a sequence of skills and concepts that children acquire as they build math knowledge. It effectively defines the developmental prerequisites for a skill or concept.

For grouping outcomes within WWC reviews for this practice guide, the panel defines a **domain** as a group of outcomes related to a child's math achievement. For this practice guide, the panel has identified six domains into which all outcomes are grouped: general numeracy, basic number concepts, number recognition, operations, geometry, and patterns and classification. The domains are not meant to be synonymous with any early math content area (see **early math content areas**).

E

Early math content areas are the specific math topics the panel believes should become the foundation of preschool, prekindergarten, and kindergarten curricula. The panel has identified number and operations, geometry, patterns, measurement, and data analysis as critical to children's math learning. Outcome domains defined for grouping outcomes in WWC reviews cover the range of skills within the early math content areas, but in some cases, the skills are grouped slightly differently (see **domain**).

F

Formal representations are the typically school-taught standard mathematical terms and symbols that represent mathematical ideas. **Informal representations** are familiar everyday objects, pictures, or words that stand for those ideas. **Informal units**, a type of informal representation, are non-standard forms of measurement, such as blocks or children's hands and feet. By contrast, examples of formal or standard measurement tools include rulers and scales. **Informal methods** are children's self-invented strategies to solve mathematical problems, and these may be supported and encouraged by teachers.

I

The **increasing magnitude principle** is the idea that a number word later in the counting sequence represents a larger quantity than a number word earlier in the counting sequence.

M

Math knowledge is a child's understanding of math concepts and skills. **Math achievement** refers to a child's performance on a variety of math tasks, including assessments.

A **multi-component intervention** is a set of instructional practices that are implemented together and evaluated as a set.

N

A **non-example** illustrates what a concept is not. For example, whereas five and six come after four and are examples of numbers larger than four, two and three come before four and are not larger. Non-examples are teaching tools designed to illustrate the difference between two things, and thus to help children learn the boundaries of a concept.

Number refers to a system for representing quantity. **Number knowledge** consists of an understanding of numbers and the relations among them. It includes the ability to recognize quantity, count, identify numerals (written numbers), and perform number operations.

Number-after knowledge is a counting skill that comes from experience with the number sequence. Children with number-after knowledge are able to identify the next number in the counting sequence without starting the count from one.

A **number list** is a series of numerals beginning with 1 and ordered by magnitude.

Number sense refers to a person's general understanding of number and operations along with the ability to use this understanding in flexible ways to make math judgments and to develop useful strategies for solving complex problems.[150]

Numerals, or written numbers, are symbols that represent numbers. For example, the numeral 8 is the symbol that represents the number eight.

O

The **one-to-one counting principle** refers to understanding one-to-one correspondence; that is, when counting, each item in a collection must be assigned one and only one number word.

The panel uses the term **operations** to refer to arithmetic. Addition and subtraction are examples of operations.

P

Prekindergarten (Pre-K) refers to the year before children enter kindergarten, usually when children are 4 years old. **Preschool** refers to the year before the prekindergarten year, when most children are 3 years old.

Progress monitoring is a systematic approach to assessment with the goal of improving skills. Progress monitoring begins with an evaluation of the child's current level of knowledge. Changes in the child's skills are then tracked through regular assessment, and goals and teaching strategies are adjusted based on the child's progress.

S

Subitizing refers to a child's ability to immediately recognize the total number of items in a collection and label it with an appropriate number word. For example, subitizing enables a child to see a collection of three toy animals and immediately know, without counting, that there are three.[151] This ability is also known as small-number recognition.

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