> we are upsetting the emotional balance. There may be circumstances in which it is not unwise to cling to illusions, but in science, we need a very different attitude, the *inductive attitude*... It requires a ready descent from the highest generalizations to the most concrete observations. It requires saying "maybe" and "perhaps" in a thousand different shades. It requires many other things, especially the following three:
>
> > **Intellectual Courage:** we should be ready to revise any one of our beliefs
> >
> > **Intellectual Honesty:** we should change a belief when there is good reason to change it
> >
> > **Wise Restraint:** we should not change a belief wantonly, without some good reason, without serious examination (p. 7–8).

To be deeper learners, students will need to *change their beliefs* about what academic work entails and about their ability to do that work. Given what we know about adolescents, such a change is likely to "upset the emotional balance." Given what we know about educational institutions, it's clear that few schools are prepared either to throw students off balance in this way or to help them regain a firmer footing.

In highly tracked schools, for example, it is very hard to challenge the assumption that some students just can't learn. When test scores deem students "below basic," it is difficult to help those students redefine themselves as competent learners. Teachers like Ms. B not only have to push back on students' prior classroom experiences, but also must challenge the messages about learning and competence that stratified school systems broadcast every day. Deeper teaching is much more than what an individual teacher does in an individual lesson, but everything he or she does in that lesson is essential to supporting deeper learning.

ENDNOTES

1 Students move toward an identity that psychologist James Greeno calls "intellective." (See Boaler & Greeno 2000.)

2 See also the *High School Survey of Student Engagement*. Consistently, each year from 2006 to 2009, 65 percent of students reported being bored everyday and only 2 percent reported never being bored. Also consider 2014 dropout statistics, which take us into the inequities in students developing "academic identities." Graduation is a low bar but suggestive of how hard it would be to achieve at a higher level (U.S. Department of Education 2012).

3 Philip Jackson, Willard Waller, and Dan Lortie were early identifiers of these patterns in the practice of teaching (Jackson 1968; Waller 1932; Lortie 1975).

4 In the Common Core State Standards for Mathematics (2010), what students need to learn about rate of change and slope are connected thus: "*Construct a function to model* a linear relationship between two quantities. Determine the rate of change and initial value of the function *from a description of a relationship* or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values." (8.F.B.4) They are also expected to "Use similar triangles to *explain why* the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; *derive* the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b." (8.EE.B.6; emphases mine.)

5 E.g., option traders study the relationship between the rate of change in the price of an option relative to a small change in the price of the underlying asset, known as an options delta; mapmakers decide on scale depending on what they want to highlight for the user; and coaches train long distance runners based on knowledge of when it makes sense to slow down or speed up in a race.

6 On this kind of graph, there is a unique pair of numbers associated with every point on an infinitely large flat surface (its *coordinates*). The coordinates describe where that point is in relation to a reference point called the *origin*: How far left or right of the origin is the point? How far above or below? Invented in the 17th century by Descartes, this representation caused a revolutionary leap in the growth of mathematics because it made possible a link between the two separate fields of algebra and geometry. It allowed an easy visual comparison between functions. It played a crucial role in the invention of calculus.

7 Ms. A is an archetype, not a real person. What she does in this lesson is a composite of the many lessons of this sort I have observed in high schools in the last five years.

8 Ms. B is a real person. The lesson we see was planned by two Residents in the Boston Teacher Residency Program (Clarissa Gore and Meaghan Provencher) and taught by one of them (Meaghan).

9 The ways that functions are represented (graphs, verbal descriptions, tables, and equations) and how the elements of these representations are connected is at the heart of the "core" mathematics in this domain. (See Leinhardt et al. 1990.)

10 Staples (2007) identifies such co-construction as an essential learning practice in secondary mathematics classrooms that seek to promote students' mathematical understanding and engagement.

11 "Look for and make use of structure" is one of the eight mathematical practices that the Common Core State Standards require teaching throughout grades K-12. But it has long been identified as a key to doing mathematics of all sorts. See for example, Kline (1972).

12 The design Ms. B is enacting is based on a protocol created by Grace Kelemanik and Amy Lucenta for the Boston Teacher Residency.

13 They are learning to communicate in what has been called "the mathematics register" by Halliday (1978). He referred to "the discipline-specific use of language employed in mathematics education" (Jablonka 2013, p. 51) as "the mathematics register." It should be noted that

this does not solely refer to specific vocabulary, but also to meanings, styles, and modes of argument.

[14] Individual development toward these competencies cannot be understood without reference to the social context within which they are embedded. See, for example, Wertsch (1988, 1985), Rogoff (1990), and Sfard (2008).

[15] Student names are pseudonyms.

[16] See the "implications for teaching" chapters in the NRC's *How People Learn: How Students Learn: History, Mathematics, and Science in the Classroom* (National Research Council 2004). For additional summaries of the implications of learning research intended to influence the design of teaching, see National Council of Teachers of Mathematics (2014) and Swan (2005).

[17] This short summary of the knowledge, skill, and commitments needed for deeper teaching is similar in content to the list of Mathematics Teaching Practices issued by the National Council of Teachers of Mathematics (2014) to align instruction with the learning goals in the Common Core State Standards (*Principles to Actions*) and the Design Principles for Instruction underlying the lesson guides issued by the Mathematics Resource Assessment project as part of the Math Design Collaborative initiated by the Bill and Melinda Gates Foundation.

[18] For example, see Fountas & Pinnell (1996).

[19] Some of the IAs in use across subjects and grade levels are collected on a website called Teacher Education by Design. They can be viewed at TEDD.org.

[20] The use of Instructional Activity protocols to enable teachers to teach mathematics ambitiously is based on research conducted by the Learning Teaching Practice project. See Lampert & Graziani (2009) and Lampert et al. (2010).

[21] This derives from Aristotelian ethics and psychological research on the acquisition of habits. For a contemporary perspective on this argument see Horn (2012).