and witness diverse approaches while being prepared to clarify the students' thinking and/or to revise the task if needed. In the example below from Ball and Bass (2000), Ball was prepared for a child's typical mathematical conception about decimals and place value and made a claim about the child's thinking based on the question asked:

> *[Ball] knows that they will often confuse .5 with .05 and that they draw this confusion, in part, from their prior conviction that 5 and 05 are the same number . . . This means that a fifth-grade teacher needs to understand a lot about the base ten number system and about positional notation. When a fifth grader asks, "Where's the 'oneths' place?" a teacher needs to be able to hear that this likely emanates from a 10-year-olds' reasonable expectation that if there is a ones place to the left of the decimal point, and a tens place to the left of that, there should be a symmetry to the right of the decimal. (p. 87)*

The numbers .5 and .05 were not randomly selected in the quote above; Ball purposefully selected these numbers to elicit students' confusion about the base-ten number system. Because Ball began with what students knew about the two numbers, she used that information to engage her students in a discussion about place value. Ultimately, if students need to have more equitable opportunities to participate in mathematics classrooms, then the students should also have the right to voice when they need support and guidance, without fear of judgment or ridicule (Boaler & Dweck, 2016).

Right 2: You Have the Right to Claim a Mistake

Closely aligned with the first RotL, the second RotL argues that students should not only have the right to claim that they are confused but have the right to claim a mistake or hold an inaccurate mathematical conception. The second right draws on the extensive research regarding the role that mathematical errors play when students learn mathematics and when teachers assess students' mathematical thinking.

Making mathematical errors is part of the learning process, especially for children who are beginning to establish the foundations of their conceptual and procedural knowledge of mathematics (Bray, 2013; Hiebert et al., 1997; Van de Walle et al., 2015). Specifically, errors can arise for different reasons: from careless computational errors arising from an oversight to what Schoenfeld (1987) described as "the result of systematic misapplications or misgeneralizations of procedures that students have learned" (p. 29). By allowing children to claim a mistake while solving problems, children explore for themselves the boundaries and assumptions of their own understanding about mathematics. Hiebert et al. (1997) argued:

> *Mistakes must be seen by the students and the teacher as places that afford opportunities to examine errors in reasoning, and thereby raise everyone's level of analysis. Mistakes are not to be covered up; they are to be used constructively. (p. 9)*

Sometimes mistakes and misconceptions are purposefully introduced by the teacher: Teachers may intentionally write an erroneous mathematical expression so that the students can reason about why such an error is incorrect (Hiebert et al., 1997). Teachers who are prepared to anticipate student responses (which include potential mistakes) can also help students see the larger landscape of mathematics and can serve as guides during instruction (Chapin, O'Connor, & Anderson, 2009; Stein et al., 2008). Teachers can also use mistakes (made by the teacher and/or claimed by the students) to inform a divergent formative assessment that sparks debate and challenge of ideas. The second RotL is supported by the Common Core Standards for Mathematical Practice (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010) in the following practice:

> *Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. ("CCSS.MATH.PRACTICE. MP3 Construct viable arguments and critique the reasoning of others")*

Furthermore, the practice of "Use Appropriate Tools Strategically" states that students should "detect possible errors by strategically using estimation and other mathematical knowledge" (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). When teachers value mistakes as insightful elements of the learning process, the mathematical practices as stated in the Common Core is evident. But before students can feel safe to share their mistakes and thinking in the classroom, teachers need to promote a safe space in which everyone should have *this right* to claim a mistake and to share this mistake with others. Furthermore, when teachers use student mistakes as formative assessment, they also can highlight the nuances of students' mathematical thinking and afford more students the opportunity to participate in the learning process, not just those who are always correct.

Right 3: Have the Right to Speak, Listen and be Heard (e.g., Engage in Conversations, Ask Questions, Share Ideas, and Listen to the Thinking of Others)

Children communicate their thinking in a variety of ways, such as through speech, writing, and body language, to name a few (Piaget, 1959; Vygotsky, 1987). Students learn how mathematics itself is a very specialized language that involves terminology and names for numbers, symbols, and operations (Gutiérrez, 2002). For students who are learning mathematics in a language other than their native language, they face many challenges to learn mathematics. As the number of emerging bilinguals increases over time, research and policy should continue to address and foreground the needs of these students as they learn mathematics (Civil & Planas, 2004; Gutiérrez & Irving, 2012; Nieto, 2013). (Note: I prefer to use the term *emerging bilinguals* in order to be more inclusive to the diversity and multiplicity of new languages that students are learning [Nieto, 2013]).

In 2015, 9.2 percent of all students enrolled in public school in the United States were designated as English language learners (Kena et al., 2015). Teachers who resist a deficit perspective of their

students, not matter the students' native language, believe every student brings a wealth of knowledge, experiences, and skills that they can use to learn mathematics (Moll et al., 1992; Nieto, 2013). Furthermore, there is research that supports the notion that children should not need to be fluent in English before they can be successful when learning mathematics (Civil, 1994; Gutiérrez, 2002; Khisty & Chval, 2002; Moschkovich, 1999). For example, a child who is a native Spanish speaker can leverage the phrase por ciento, or "per 100," to convert fractions into equivalent percentages out of 100.

Because emerging bilinguals take on average seven years to develop fluency in another language (Cummins, 2008), the right to verbally communicate and to be heard while engaging in mathematical thinking is especially crucial for them. The third RotL also helps teachers create more opportunities to measure the language and mathematical proficiency of emerging bilinguals in their classroom while all children exercise their right to communicate their thinking and listen to the thinking of others.

Other ways that the third RotL promotes equity and supports students' mathematical thinking is through the act of revoicing (Chapin et al., 2009; Herbel-Eisenmann, Drake, & Cirillo, 2009; Kazemi & Hintz, 2013; O'Connor & Michaels, 1993; Shein, 2012). Revoicing helps teachers to:

> *(1) position students in differing alignments with propositions and allow them to claim or disclaim ownership of their position; (2) share reformulations in ways that credit students with teachers' warranted inferences; (3) scaffold and recast problem-solution strategies of non-native-language students. (O'Connor & Michaels, 1993, p. 318)*

For example, Chapin et al., (2009) presented the case of Phillipe, who suggests that 24 is an odd number. When the teacher asks the students to restate what Phillipe said in their own words, the students have "more time to process Phillipe's statement," and this "supports the teachers' goal of giving all students full access to participation" (Chapin et al., 2009, p. 2). Revoicing affords students an opportunity to learn from each other while exercising their third right of the learner.

There is caution to not assume that revoicing is simply repeating someone else said; instead, revoicing is much more than that. Teachers can use a revoicing strategy to "clarify, amplify, or highlight an idea" (Kazemi & Hintz, 2014, p. 30), especially when students are confused or express a mistake. Returning to the Common Core State Standards of Mathematical Practice, if students are expected to "construct viable arguments and critique the reasoning of others," then they need to utilize their right to verbally communicate their thinking with others, even if that thinking might be imperfect at the time they share (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). Furthermore, when teachers foreground the third RotL, students can agree and/or disagree with the ideas presented by another, which serves as another snapshot into students' mathematical thinking (Reinhart, 2000).

Right 4: You Have the Right to Write, Do, and Represent Only What Makes Sense to You

If students have the right to share their ideas and listen to each other's thinking, then it follows that they should also have the right to write; do (model with gestures and manipulate with tools); and represent what makes sense to them. There is no one way to "do mathematics" and/or represent one's thinking in written work with symbols, pictures, and representations. Teachers can learn a great deal about mathematical thinking and understanding from children's multiple mathematical representations (Carpenter, Fennema, & Franke, 1996; Carpenter, Fennema, Loef Franke, Levi, & Empson, 1999; Empson & Levi, 2011; Fennema, Franke, Carpenter, & Carey, 1993; Kazemi & Franke, 2004; Philipp, Clement, Thanheiser, Schappelle, & Sowder, 2003).

When students have the right to write, do, and represent what makes sense to them, students are encouraged to find multiple ways in which to justify their thinking and solution strategies. When students have an opportunity to represent what they know first, students' existing knowledge is pushed to the forefront, which can promote productive discussions amongst students and teachers (Kazemi & Hintz, 2013; Parrish, 2010). As Kazemi and Loef Franke (2004) have argued, teachers who elicit and make sense of students' mathematical thinking through "student work also allowed the teachers' to begin to see themselves as mathematical thinkers when they were willing to struggle through student strategies they did not understand" (p. 230). Kazemi & Loef Franke's quote rings true when we consider that teachers can also exercise their first RotL (to be confused) when they encounter unfamiliar student strategies that arise out of divergent formative assessments.

The third and fourth rights honor the sociocultural nature of learning, doing, and teaching mathematics in the world (Atweh, Forgasz, & Nebres, 2001). Although many traditional mathematics textbooks rarely address the intersection of culture and mathematics, others have written extensively about the ways in which culture, language, and social practices inform the field of mathematics (Civil, 2002; D'Ambrosio, 1990; Gutstein, Lipman, Hernandez, & de los Reyes, 1997; Nasir, Hand, & Taylor, 2008; Orey, 2011; Turner et al., 2014). Specifically, the field of ethnomathematics argues that culture and mathematics are inextricably tied because of how we live, interact with each other, learn new knowledge, and make sense of our environments in the world (Barta, Eglash, & Barkley, 2014; Barton, 1996; Borba, 1990; D'Ambrosio, 1990; Zaslavsky, 1998). When considering the diverse ways that people across the world have developed mathematical ideas with the symbols and terminology to express these ideas, the fourth RotL acknowledges that each student in a classroom may bring a way of expressing mathematical thinking in a written format that is specific to his or her culture, background, and experiences, and teachers should learn how to honor this knowledge.

The right for all students to do mathematics (including gestures and manipulation of tools) and to represent what makes sense to them (with pictures and written work) is even more important when the wealth of knowledge and resources that immigrants bring to the classroom is considered (Orey, 2011;

Perkins & Flores, 2002; Philipp, 1996). For example, Moschkovich (2013), a scholar in the field of equity for emerging bilinguals in mathematics classrooms, has noted how "in some countries a period is used for marking the thousands place, not for decimals as in the United States (writing 1.234 instead of 1,234), and the comma is used to mark decimals (writing 10,03 not 10.03)" (p. 29). A teacher who is not prepared to recognize the difference of notation highlighted in Moschkovich's example may see the use of the comma as a sign of careless or sloppy notation, when in fact the use of the comma is a valid notation in other parts of the world. When teachers foreground students' diverse ideas, background, and experiences (many of which are tied to their language and culture), they can use authentic written formative assessments that begin with what students already know as a learning opportunity about their students' mathematical thinking.

Mathematics Teacher Educators, Prospective Teachers, and the RotL

When I first heard about Torres's Rights of the Learner, I was struck by the simplicity of the rights, but more importantly, the explicit powers afforded to the students. As a mathematics student, my teachers would say to me that that "mathematics was supposed to be hard" and that "the error you made is a common one I see by many other students who are learning this material." But what I rarely heard was how my confusions and errors were *my right as a learner* in the classroom. As I reflected about my prior experiences as a mathematics student and as a high school and middle school mathematics teacher, the RoL fundamentally shifted my perception of how I learned and taught mathematics.

As a mathematics teacher educator who prepares new elementary teachers, I see how the RotL plays a role in my perception of what it means to know mathematics. It is my responsibility to help my new teachers see how mistakes, as a form of "rough draft talk" (Jansen, Cooper, Vascellaro, & Wandless, 2016), should not be avoided but instead valued as glimpses into students' thinking *at that moment*. Therefore, I frame my elementary mathematics methods class as opportunities for my prospective teachers (PTs) to adopt the RotL both for themselves as they learn to teach mathematics and for their students they encounter in their fieldwork. In the following section, I describe how I help my PTs to engage in an assignment called a case study of a child's mathematical thinking (Empson, Junk, & Turner, 2006; Philipp et al., 2003; Turner et al., 2012) as an opportunity to help children exercise their rights as learners.

A cornerstone of my practice as a mathematics teacher educator is to help my PTs plan and implement a mathematics lesson by beginning with what children already know about mathematics. My PTs learn about divergent formative assessments that elicit children's mathematical thinking, and the case study of a child's mathematical thinking (TeachMath, 2016) is one of the first assignments that I give to accomplish this goal. Based on the extensive work of the TEACH Math (Teachers Empowered to Advance Change in Mathematics) research group, the case study is framed as a series of problem-solving interviews that elicit children's Funds of Knowledge (Moll et al., 1992) and children's mathematical thinking (Carpenter et al., 1999) about the operations, base ten knowledge, and rational numbers.

In the first interview, the getting to know you interview, the PTs pose a series of questions to a specific child in their field experience classroom about their interests, beliefs, and perceptions about mathematics, and potential home and community practices that could serve as a resource when designing mathematics tasks. Example questions[2] the PTs have asked their case study child are as follows:

> *"Where do you like to go with family/friends? What are some places in the community that you like to go to with your family? What do you do there? For example, where do you like to go on the weekends with your family? This can include places such as grocery or other shopping . . . Can you think of any places in your community where people do math or use math? What about your family members—where do they use math? Where do they do math? . . . Have you learned math in a different school? Country? How was it similar or different?" (TeachMath, 2016, p. 8)*

While conducting this interview, the PTs learn about the child's and family's out-of-school practices, perceptions about mathematics, and community resources that help them contextualize mathematics tasks they prepare as a part of my course.

The remainder of the case study assignment asks PTs to conduct a series of problem-solving interviews (Ginsburg, 1997; Ginsburg, Jacobs, & Lopez, 1998; Ginsburg, Jang, Preston, VanEsselstyn, Appel, 2004) with their case study student. I use interviews that follow a sequential, adaptive-learning format (Empson et al., 2006) so that students who answer correctly are given more challenging problems. Throughout each section in the interviews, PTs are expected to ask probing questions that clarify students' mathematical thinking (Jacobs & Ambrose, 2008). As detailed in the assignment guidelines for the interviews, the PTs foreground the child's mathematical thinking in the interviews:

> *The purpose of this interview is to learn more about how your student solves a series of mathematical problem solving tasks. This is your opportunity to learn how children solve math* without a teacher's intervention or explicit guidance *. . . Ultimately, the goal of this interview is not for your child to get all of them correct; instead, your responsibility is to learn and absorb as much as you can about your case study's strategies for solving the tasks and how you can improve your technique of supporting, clarifying, and extending their mathematical thinking. (adapted from Empson et al., 2006 and TeachMath, 2016, p. 15)*

The PTs leverage what they have learned about using formative assessment to elicit children's mathematical thinking and the role of appropriate number choice as they plan and implement their interviews. After the PTs conduct the interviews, they analyze the student responses for an understanding of base ten and any insights they gleaned during the interviews. The PTs are encouraged to analyze the child's interview responses in the lens of the RotL: The problem-solving interviews serve as a safe space for children to share their confusions and mistakes to have their thinking valued by the teacher. The interviews are not intended to

become tutoring sessions where PTs correct student errors and/or help students memorize vocabulary and key words in the problems they solve. Instead, the problem-solving interviews help PTs elicit students' "rough draft talk" (Jansen, Cooper, Vascellaro, & Wandless, 2016) as thinking that is under constant revision and clarification through more iterations of learning and discussion. Although many of my PTs have adopted the RotL as they learn to promote equity in their mathematics instruction by foregrounding divergent formative assessment, I have faced numerous complexities and challenges as a mathematics teacher educator.

Complexities with the Rights of the Learner

Every semester that I teach elementary mathematics methods, I have noticed that sometimes the RotL comes in direct conflict with my own beliefs and philosophy for teaching mathematics and with the authentic situations my PTs encounter in the field. In one such situation, some of my PTs shared the following stereotypes and biases about children and families who live in communities designated as low-income, who identify as Latinx, and/or who are recent immigrants to the United States:

- This neighborhood around the school probably has a lot of gang activity; I should probably keep a close eye on my car.
- These parents just don't care about education; I don't ever see them volunteering at school.
- Maybe they should learn English better before we teach them mathematics.

When I hear these comments, I remind myself that the comments are made without evidence or fact and are rooted in their assumptions of communities, families, and children from backgrounds that PTs may not be familiar with. Nonetheless, I face an internal struggle with these comments and the RotL: Should teacher educators still give PTs the right to say something that might marginalize a child or their family even if sharing these comments could be a first step toward safely uncovering and addressing their dormant stereotypes and assumptions? My initial reaction is of sadness and frustration because I have seen the direct impact that stereotypes and assumptions can have for children, families, and communities who have been marginalized in the past.

After I shared with Torres the struggles I faced to help my PTs adopt the third RotL (to speak, listen and be heard), she stated:

> *One of the things that I encounter is that teachers will buck. Because you're asking them to relinquish a cultural experience of education. And what we're promoting is a paradigm shift and you're challenging conventional wisdom. . . . But it's a cultural shock and they're so used to school being a certain way, what we're trying to promote is a defiance towards conventional wisdom . . . It's not just about teaching, but that they are researchers, and they need to accept that what they know is tentative and can be changed at any given time based on new information that challenges their existing viewpoint. (Torres, personal communication, March 7, 2016)*

Torres's insight spurred a moment of meta-reflection for me about the RotL in two ways: The RotL can help PTs learn how to adopt asset-based thinking about teaching mathematics and can help mathematics teacher educators adopt a nuanced perspective about their PTs. I have seen firsthand my PTs "unlearning how to teach mathematics" (Ball, 1988) as we use the RotL to reframe their prior experiences and to learn new, more equitable approaches to teaching mathematics. Similarly, I have learned from the conversation with Torres that my PTs are in a constant state of flux with their thinking about children who may come from backgrounds that they are unfamiliar with. The RotL is a risky space for me as a mathematics teacher educator: As I open safe spaces for my PTs to explore their stereotypes, assumptions, and biases about the children that they encounter in the field, I should support them to question and revise their thinking about children and families they will serve in the future. Furthermore, I recognize that not all of my PTs will consistently adopt a pedagogical stance that foregrounds social justice and equity at the conclusion of my course, but eventually they may, after graduation when they have their own classrooms. Nonetheless, Torres and others (Aguirre, 2009; Wager & Stinson, 2012; White, Crespo, & Civil, 2016) continue to (re)frame my work in teacher education as an intermediary moment in my PTs' journey to become a teacher. As Torres has claimed, PTs who learn how to incorporate the RotL into their mathematics instruction can continue to revise their thinking about promoting equity in the classroom:

> *If you can plant the seeds of doubt, interest and curiosity, then they will, hopefully take root and overtime they will evolve. But that's all we [as teacher educators] can do. We can't change it in a semester, but we can plant the seeds. (Torres, personal communication, March 7, 2016)*

Torres's RotL and the notion of "rough draft talk" (Jansen et al., 2016) inform my practice as a mathematics teacher educator who constantly questions and critiques my own practice so that my PTs can also engage in *the same inquiry for themselves*. When teachers constantly critique and reflect about their practice and the practices of others, they enter a more honest space that can dismantle nuanced systems that perpetuate inequities in our schools and classrooms (Kalinec-Craig 2015; Kalinec-Craig & Bonner, 2016; Cochran-Smith, 1991; Ball & Tyson, 2011; Gutiérrez, 2015; Joseph, Haynes, & Cobb, 2016).

Conclusion

In this paper, I have presented the Rights of the Learner as first conceptualized by Torres and how I have interpreted and applied these rights to my practice as a mathematics teacher educator. The four RotL encourage teachers to both push children's assets and resources to the forefront of teaching mathematics and leverage divergent formative assessment as a tool to elicit the ways children know, use, and learn mathematics. Some have questioned as to whether there may be more than four RotL; I agree that there may in fact be many more rights that a teacher can develop and adopt into her practice. The purpose of this paper was to not provide a laundry list of norms that teachers could use as a checklist for promoting equity while teaching mathematics. Instead, these four RotL can serve as a beginning to a larger conversation about the

ways that teachers and teacher educators can implement strategies that promote equity in the classroom and align with existing practices, such as formative assessment, that they already incorporate to inform their practice.

The need for children to have equitable opportunities to learn and be successful in mathematics is urgent now more than ever (Gutiérrez & Irving, 2012; Kena et al., 2015). Teachers who pass on ownership of the mathematical thinking to their students also encourage students to take more risks in their thinking and to push the boundaries of what they know or assume to know about mathematics. As future teachers enter teacher preparation programs, they too need to be prepared to rethink what they know or assume to know about teaching mathematics. The RotL can be one way in which PTs see their students as citizens in a democracy who exercise their right to know, use, and communicate their knowledge of mathematics.